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The Square-Root Scree Plot: A Simple Improvement to a Classic Display

Marco Del Giudice*

OpenPsych

Abstract

Scree plots are ubiquitous in applications of exploratory factor analysis (EFA) and principal component analysis (PCA); they are used to visualize the relative importance of different factors/components and display the results of selection procedures (e.g., parallel analysis). Because the eigenvalues shown in the scree plot indicate the amounts of variance accounted for by the corresponding factors/components, they tend to give a distorted picture of the relative importance of the factors/components with respect to the original units of the variables. Specifically, variances inflate the apparent relative importance of large effects and deflate that of small effects; as a result, traditional scree plots exaggerate the differences between larger and smaller factors/components, and flatten the visual representation of the smaller ones. In this brief note, I propose a simple solution in the form of *square-root scree plots*, i.e., scree plots based on the square root of the eigenvalues. Square-root scree plots provide a balanced display of the relative importance of the factors/components, and a more legible representation of the smaller ones. They are a useful addition to the toolkit of EFA and PCA, and may be preferable as a default option in most common applications.

Keywords: Eigenvalues, factor analysis, principal component analysis, scree plot

1 Introduction

The scree plot was introduced by Cattell (1966) and remains a ubiquitous display in exploratory factor analysis (EFA) and principal component analysis (PCA). A scree plot shows the eigenvalues of the factors or components, ordered from the first (largest) to the last (smallest; Figure 1 (a)). Scree plots can be augmented with lines corresponding to selection criteria for the number of factors/components to retain, for example parallel analysis (see Lim & Jahng 2019; Revelle 2022). Since eigenvalues indicate the amounts of variance accounted for by the corresponding factors/components, scree plots are often used to visualize the relative importance of the factors/components extracted in the analysis.

This use of scree plots is problematic, because explained variance is a non-intuitive and potentially misleading index of importance. Variances are mathematically convenient because they combine additively; however, they are not expressed in the original units of the variables of interest—say intelligence in IQ points, or height in inches—but in squared units. Even when these variance units are not meaningless (as with squared IQ points), they still fail to measure the actual trait under consideration (e.g., square inches do not measure a person's height). As has been noted multiple times in the methodological literature, using the proportion of explained variance as an effect size can easily lead researchers to dramatically underestimate the real-world importance of certain effects (e.g. Abelson 1985; D'Andrade & Dart 1990; Funder & Ozer 2019).

An important corollary is that comparing effects based on variances tends to exaggerate the differences among them—often by a large margin (Del Giudice, 2021; Hunter & Schmidt, 1990). For example, consider two variables X_1 and X_2, which correlate .60 and .20 with variable Y. A given change in X_1 (e.g., one standard deviation) corresponds to a change in Y that is three times larger than the one associated with the same change

*Department of Psychology, University of New Mexico, Logan Hall, 2001 Redondo Dr. NE, Albuquerque, NM 87131, USA, Email: marcodg@unm.edu

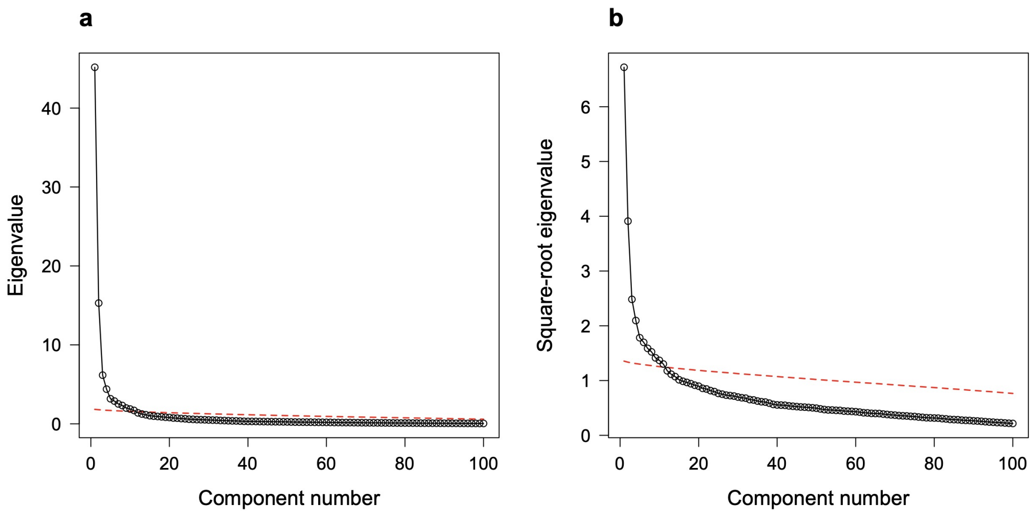

Figure 1: (a) Traditional scree plot for the PCA of a simulated dataset. The dashed red line shows the results of parallel analysis, which suggests 11 reliable components. (b) Square-root scree plot for the same analysis.

in X_2. However, the variance explained by X_1 (36 %, expressed in *squared* units of Y) is *nine times* that explained by X_2 (4 %). The general point is that variances make large effects appear comparatively larger and small effects appear smaller; this distortion becomes more pronounced as the effects being compared grow more different from each other. To properly compare the "real-world" effects of two variables (i.e., their effects expressed in the original units of the variables) one should not compare proportions of variance but rather their square roots (the correlation coefficients in the example of X_1 and X_2). In a previous paper (Del Giudice, 2021), I discussed this point in detail and reviewed some of its implications for behavior genetics and psychometrics.

Since the eigenvalues displayed in traditional scree plots correspond to amounts of variance accounted for, they produce the same kind of distortion when they are used to assess the relative importance of different factors/components. For example, the first component displayed in Figure 1 (a) accounts for about three times more variance than the second component (specifically, 40 % vs. 13 %), and visually dominates the scree plot. But in the original units of the variables, the first component explains only about 1.7 times as much as the second component ($\sqrt{3} \approx 1.73$). The third component is only one eight as important as the first with respect to the variance (5 % vs. 40 %), but about one third as important with respect to the original variables ($\sqrt{1/8} \approx 0.35$). Of course, all these comparisons are more meaningful when the variables included in the analysis are on the same scale (e.g., standardized to unit variance).

In addition to inflating the importance of the larger factors/components, traditional scree plots tend to flatten and compress the visual representation of the smaller ones. In Figure 1 (a), one struggles to make distinctions and comparisons beyond the first few components; similarly, it is difficult to see what is going on in the region where the PCA solution (solid black line) intersects the result of parallel analysis (dashed red line).

These limitations of the traditional scree plot have a simple solution: by replacing the eigenvalues with their square roots, one obtains what I will call a *square-root scree plot*. The square roots of the eigenvalues track the contribution of the factors/components in the original units of the variables, and offer a balanced visualization of relative importance. Figure 1 (b) shows the square-root scree plot for the same analysis of Figure 1 (a). The new plot does not exaggerate the importance of the larger components, facilitates comparisons between the smaller ones, and displays the results of the parallel analysis in a clear and legible way.[1] While the square-root scree plot is a straightforward, easily computed variant of the traditional scree plot, to my knowledge it has not been employed before in the context of EFA and PCA.[2] The square-root scree plot represents a useful addition

[1] Note that the classic Kaiser-Guttman criterion (eigenvalues > 1) is displayed in exactly the same way in traditional and square-root scree plots, because $\sqrt{1} = 1$. This criterion has limitations that make it less than ideal, but is still very popular in practice. Moreover, parallel analysis of PCA tends to converge with the Kaiser-Guttman criterion as sample size increases (Revelle, 2022). A possible variant of the square-root scree plot would display the square roots of the normalized eigenvalues (i.e., eigenvalues divided by their sum, corresponding to variance proportions). Doing so would increase the intuitive correspondence with correlation coefficients, but lose other desirable properties (e.g., the invariance of $\sqrt{1} = 1$).

[2] The closest example I could find is a paper by Dey & Stephens (2018), who compared correlation matrices by displaying plots of their square-root eigenvalues (see their Figure 5).

to the toolkit of these techniques and may be preferable as a default option, unless researchers have substantive reasons to be specifically interested in the variance accounted for by the factors/components.

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