Supplementary Materials
Volatile opinions and optimal control of vaccine awareness campaigns: chaotic behaviour of the Forward–Backward Sweep algorithm vs heuristic direct optimization

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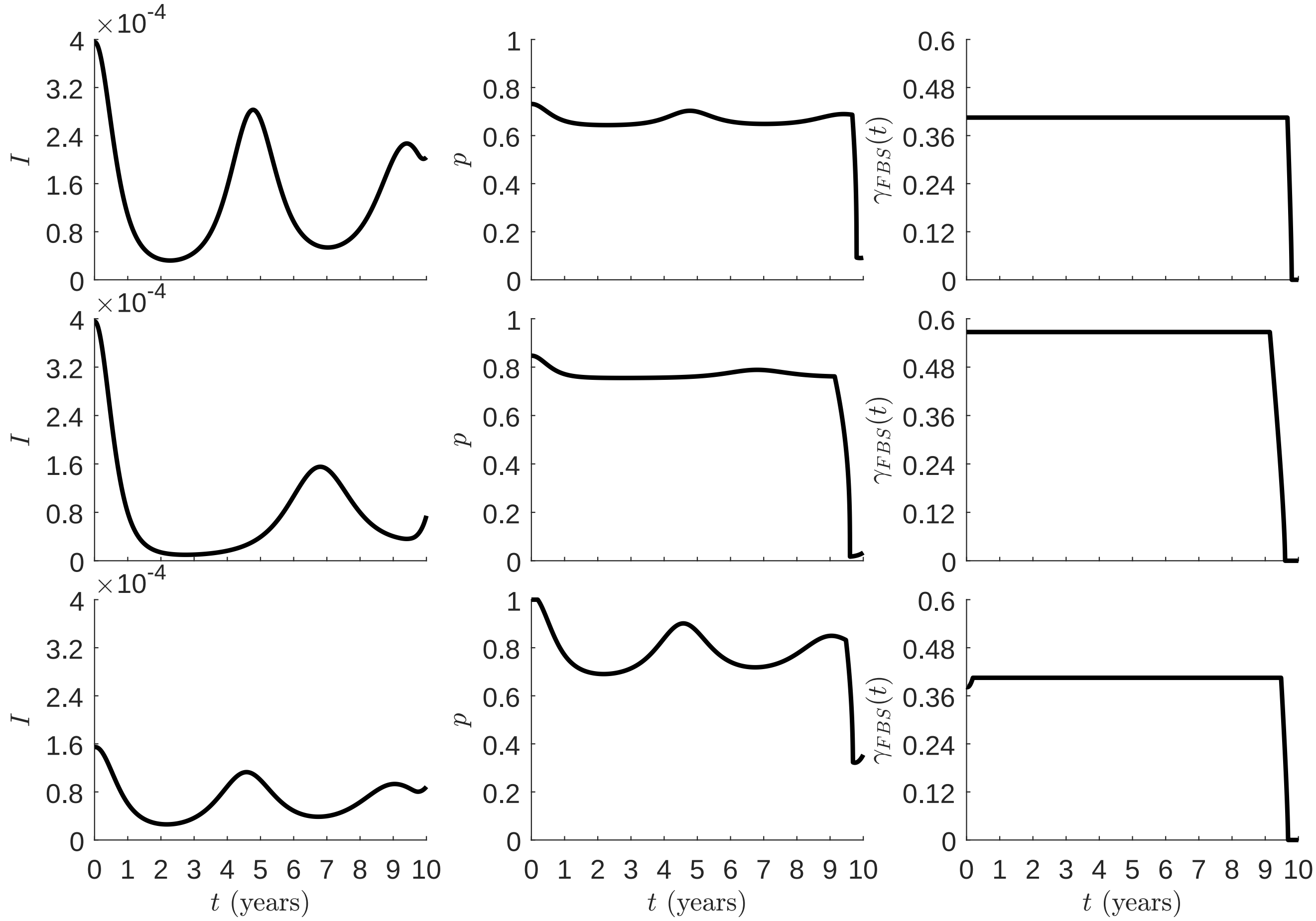

Figure S1: OC solutions by FBS method for the simulation scenarios C2 (top row), C3 (second row) and C4 (bottom row). Left panels: I; central panels: p; right panels: $\gamma_{FBS}(t)$. Initial data correspond to the endemic equilibrium of model (6) with $\gamma(t) \equiv 0$. Other parameter values are listed in Table 1.

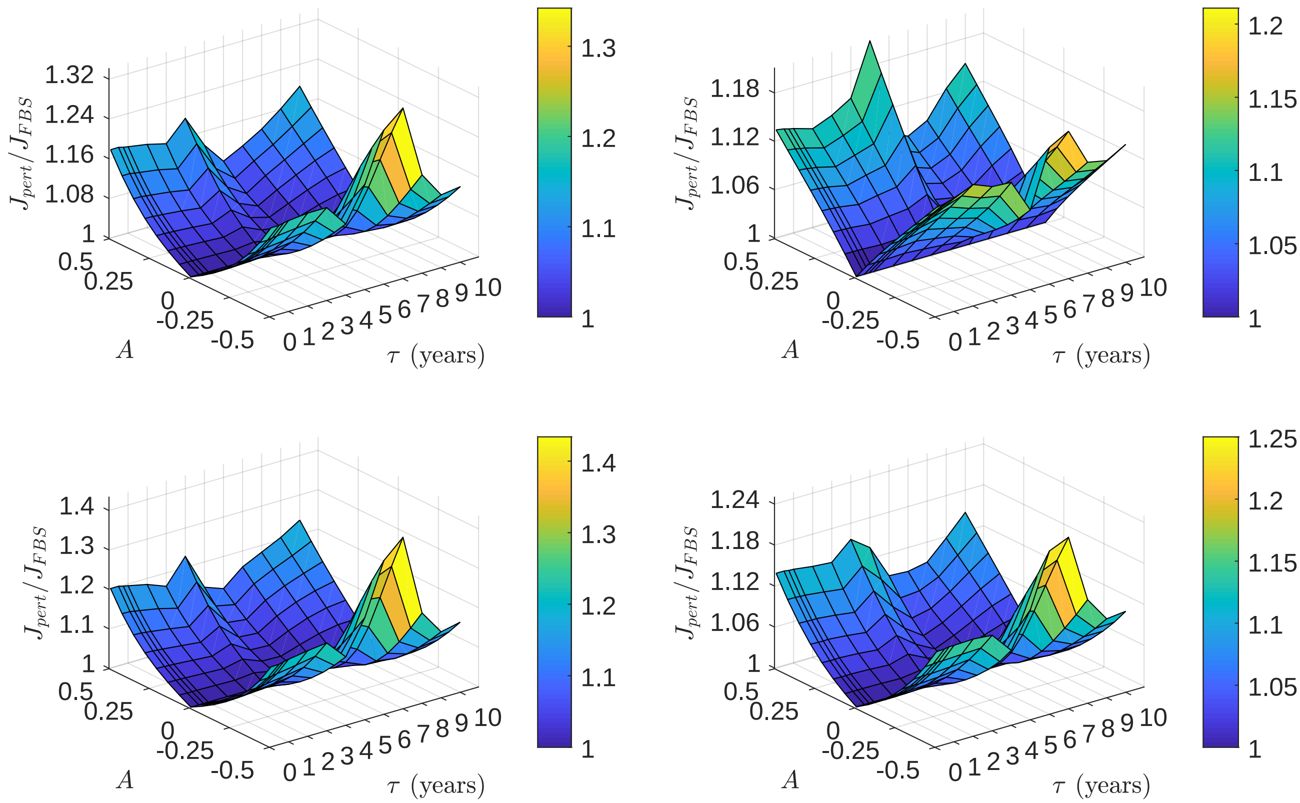

Figure S2: J_{pert}/J_{FBS} as function of A and τ for the simulation scenario C1 (top left panel), C2 (top right panel), C3 (bottom left panel) and C5 (bottom right panel). Initial data correspond to: $S(0) = 0.0999972$, $I(0) = 2.3878 \cdot 10^{-4}$ for the case C1; the endemic equilibrium of model (6) with $\gamma(t) \equiv 0$ for the other cases. Other parameter values are listed in Table 1.

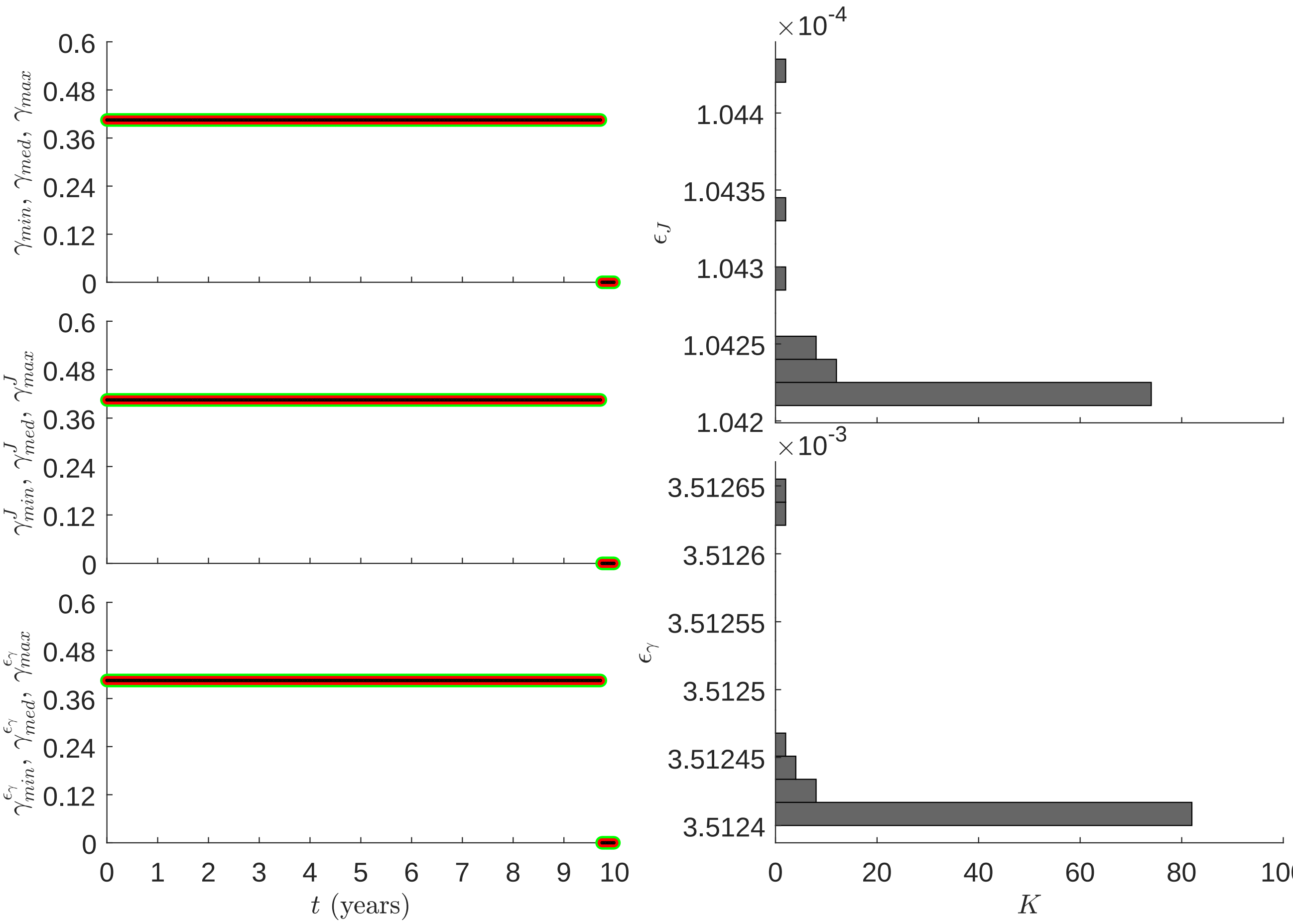

Figure S3: Statistical assessment for the case $C_\gamma = C_\gamma^{(o)}$ and simulation scenario C2. Data obtained by applying $K = 100$ times the PSO algorithm. Left top panel: $\gamma_{min}(t)$ (green line), $\gamma_{med}(t)$ (red line) and $\gamma_{max}(t)$ (black line). Left central panel: $\gamma^J_{min}(t)$ (green line), $\gamma^J_{med}(t)$ (red line) and $\gamma^J_{max}(t)$ (black line). Left bottom panel: $\gamma^{\epsilon_\gamma}_{min}(t)$ (green line), $\gamma^{\epsilon_\gamma}_{med}(t)$ (red line) and $\gamma^{\epsilon_\gamma}_{max}(t)$ (black line). Right panels: distribution of ϵ_J (top panel) and of ϵ_γ (bottom panel). Parameter values and initial data as in Fig. S1.

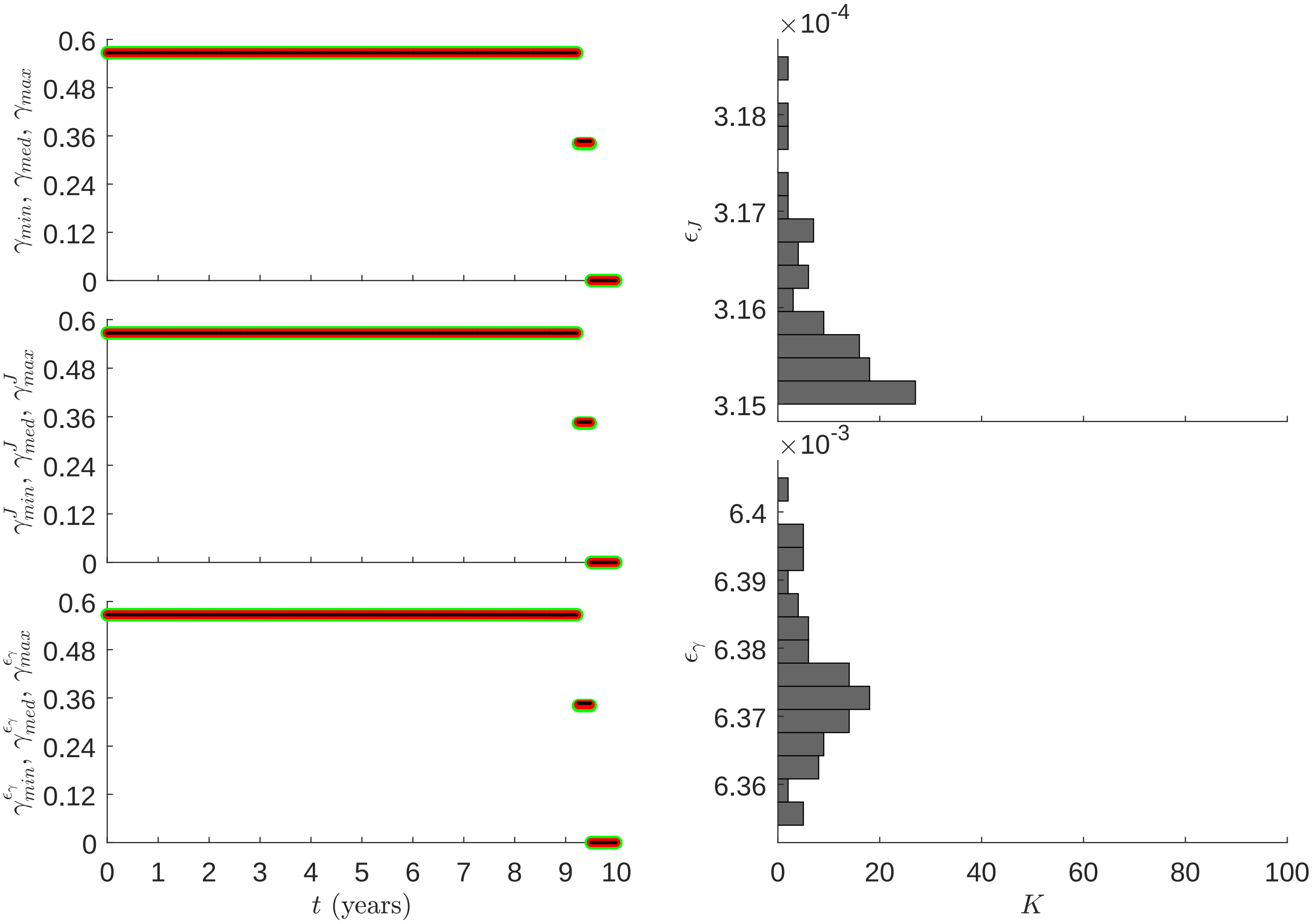

Figure S4: Statistical assessment for the case $C_\gamma = C_\gamma^{(o)}$ and simulation scenario C3. Data obtained by applying $K = 100$ times the PSO algorithm. Left top panel: $\gamma_{min}(t)$ (green line), $\gamma_{med}(t)$ (red line) and $\gamma_{max}(t)$ (black line). Left central panel: $\gamma^J_{min}(t)$ (green line), $\gamma^J_{med}(t)$ (red line) and $\gamma^J_{max}(t)$ (black line). Left bottom panel: $\gamma^{\epsilon_\gamma}_{min}(t)$ (green line), $\gamma^{\epsilon_\gamma}_{med}(t)$ (red line) and $\gamma^{\epsilon_\gamma}_{max}(t)$ (black line). Right panels: distribution of ϵ_J (top panel) and of ϵ_γ (bottom panel). Parameter values and initial data as in Fig. S1.

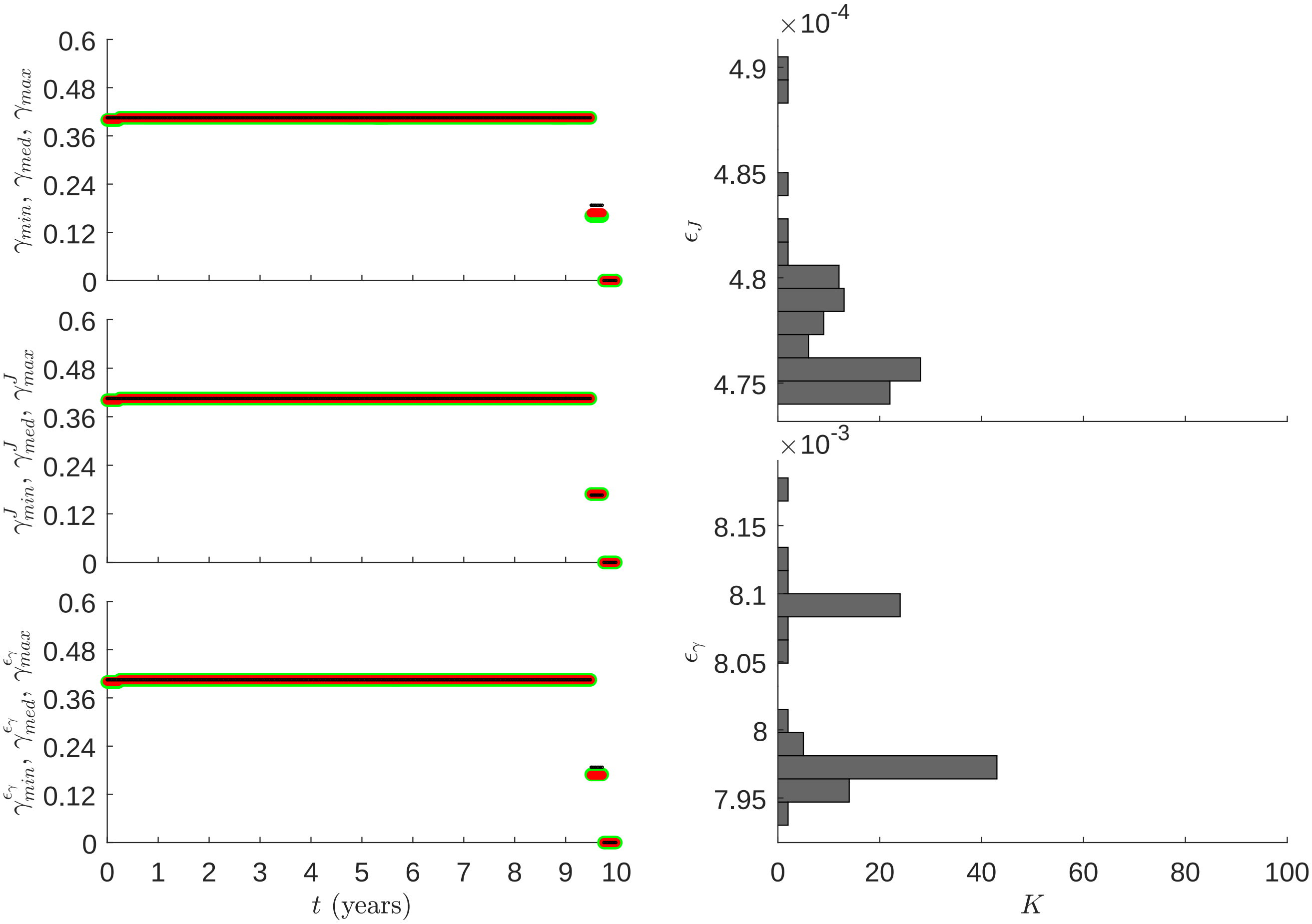

Figure S5: Statistical assessment for the case $C_\gamma = C_\gamma^{(o)}$ and simulation scenario C4. Data obtained by applying $K = 100$ times the PSO algorithm. Left top panel: $\gamma_{min}(t)$ (green line), $\gamma_{med}(t)$ (red line) and $\gamma_{max}(t)$ (black line). Left central panel: $\gamma^J_{min}(t)$ (green line), $\gamma^J_{med}(t)$ (red line) and $\gamma^J_{max}(t)$ (black line). Left bottom panel: $\gamma^{\epsilon_\gamma}_{min}(t)$ (green line), $\gamma^{\epsilon_\gamma}_{med}(t)$ (red line) and $\gamma^{\epsilon_\gamma}_{max}(t)$ (black line). Right panels: distribution of ϵ_J (top panel) and of ϵ_γ (bottom panel). Parameter values and initial data as in Fig. S1.

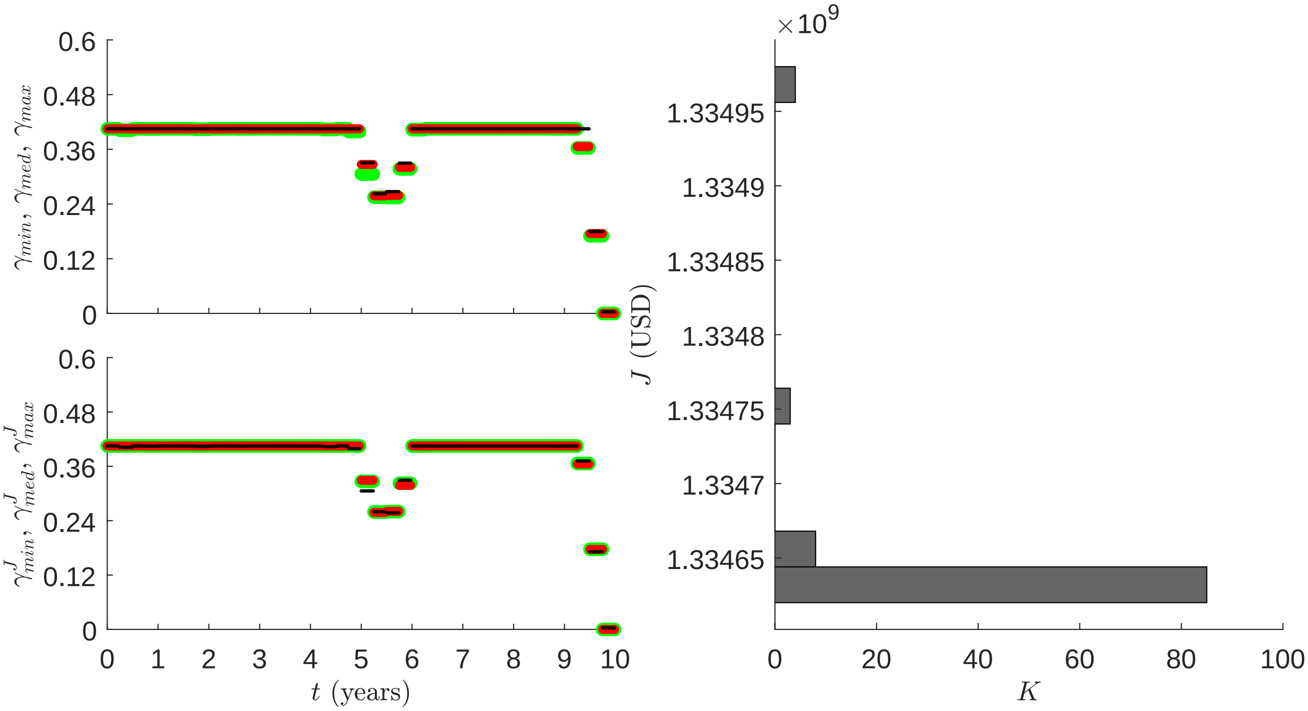

Figure S6: Statistical assessment for the case $C_\gamma = 5C_\gamma^{(o)}$ with 3 months programming and simulation scenario C2. Data obtained by applying $K = 100$ times the PSO algorithm. Left top panel: $\gamma_{min}(t)$ (green line), $\gamma_{med}(t)$ (red line) and $\gamma_{max}(t)$ (black line). Left bottom panel: $\gamma^J_{min}(t)$ (green line), $\gamma^J_{med}(t)$ (red line) and $\gamma^J_{max}(t)$ (black line). Right panel: distribution of J. Parameter values and initial data as in Fig. S1.

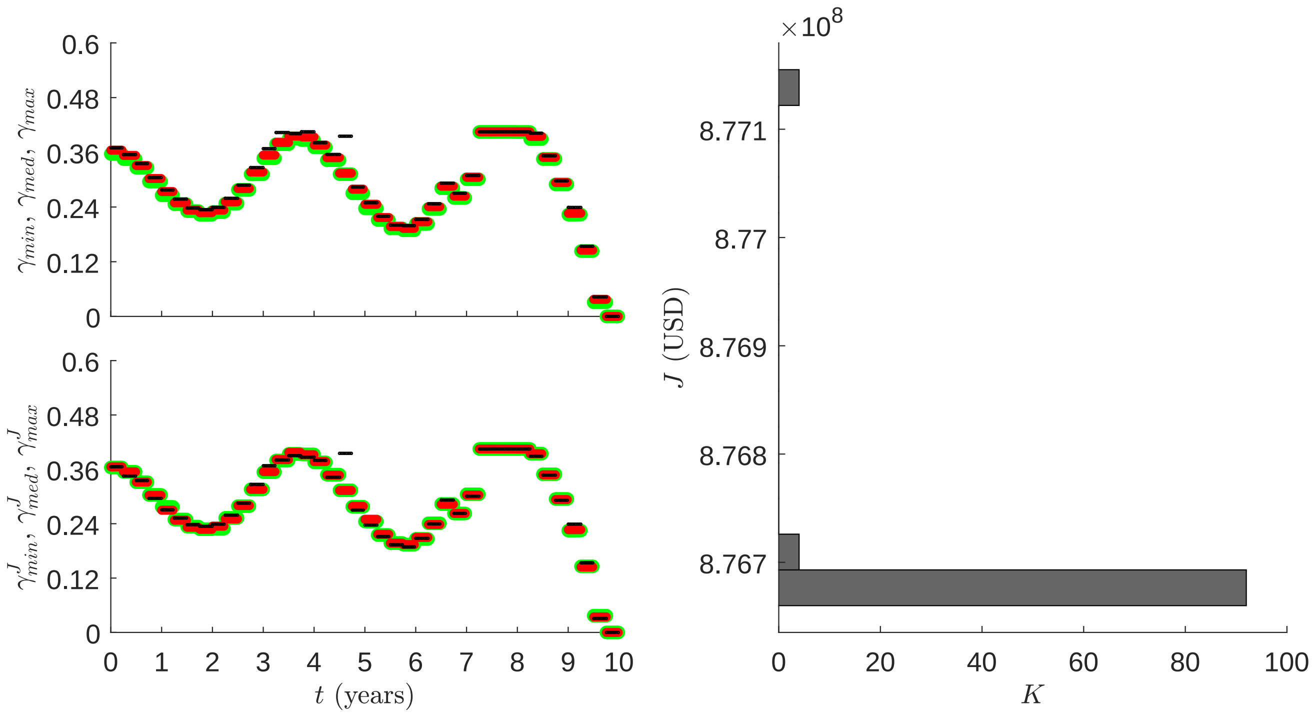

Figure S7: Statistical assessment for the case $C_\gamma = 5C_\gamma^{(o)}$ with 3 months programming and simulation scenario C4. Data obtained by applying $K = 100$ times the PSO algorithm. Left top panel: $\gamma_{min}(t)$ (green line), $\gamma_{med}(t)$ (red line) and $\gamma_{max}(t)$ (black line). Left bottom panel: $\gamma^J_{min}(t)$ (green line), $\gamma^J_{med}(t)$ (red line) and $\gamma^J_{max}(t)$ (black line). Right panel: distribution of J. Parameter values and initial data as in Fig. S1.

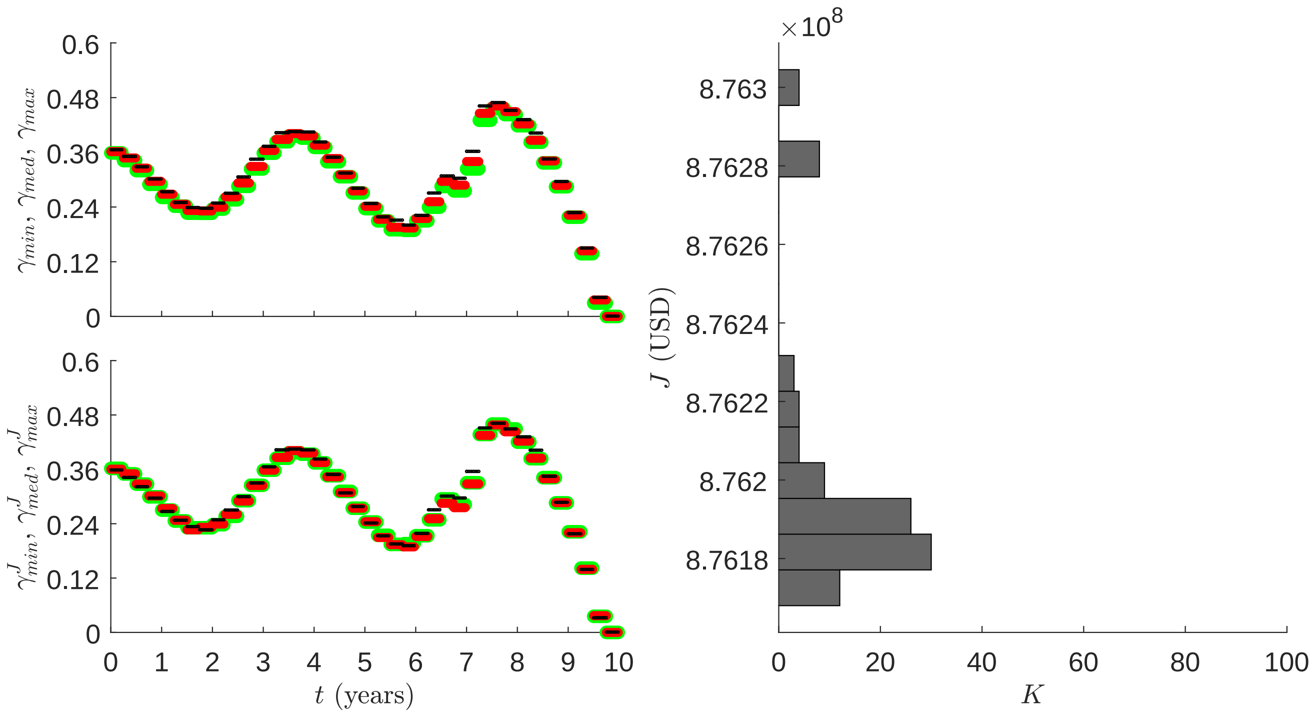

Figure S8: Statistical assessment for the case $C_\gamma = 5C_\gamma^{(o)}$ with 3 months programming and simulation scenario C5. Data obtained by applying $K = 100$ times the PSO algorithm. Left top panel: $\gamma_{min}(t)$ (green line), $\gamma_{med}(t)$ (red line) and $\gamma_{max}(t)$ (black line). Left bottom panel: $\gamma^J_{min}(t)$ (green line), $\gamma^J_{med}(t)$ (red line) and $\gamma^J_{max}(t)$ (black line). Right panel: distribution of J. Parameter values and initial data as in Fig. S2.

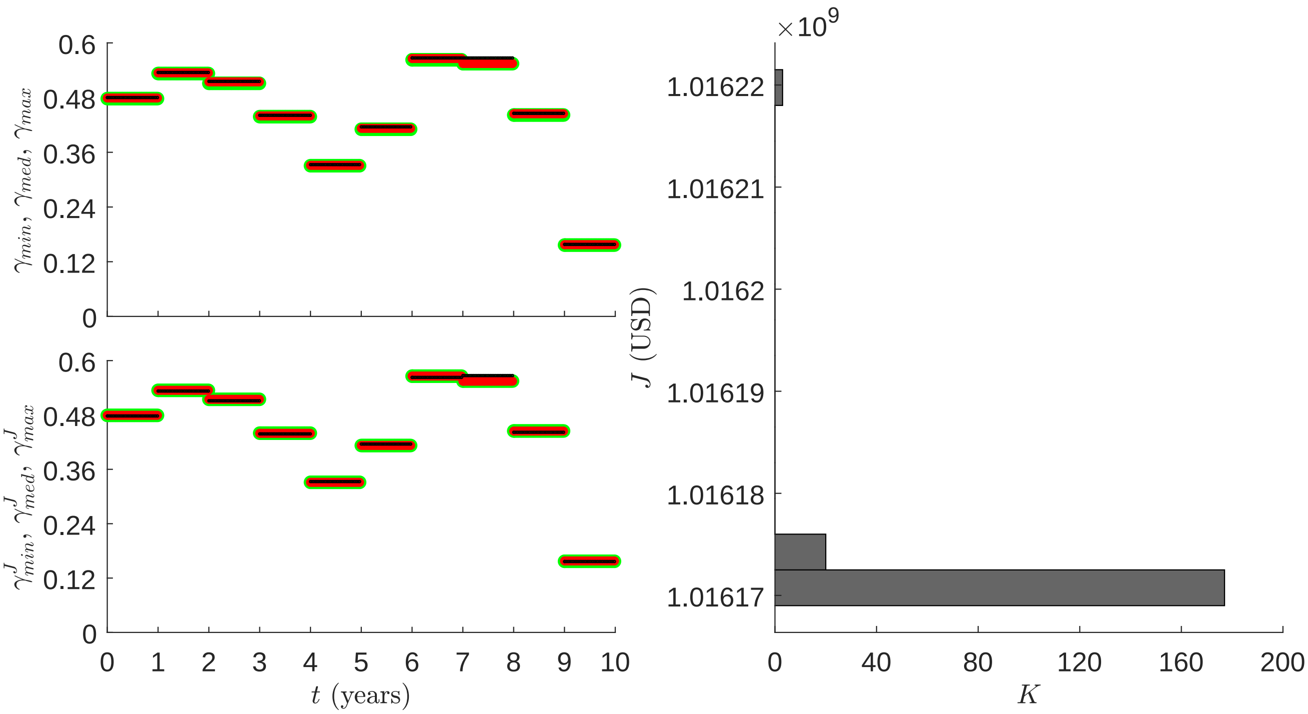

Figure S9: Statistical assessment for the case $C_\gamma = 5C_\gamma^{(o)}$ with 1 year programming and simulation scenario C1. Data obtained by applying $K = 200$ times the PSO algorithm. Left top panel: $\gamma_{min}(t)$ (green line), $\gamma_{med}(t)$ (red line) and $\gamma_{max}(t)$ (black line). Left bottom panel: $\gamma^J_{min}(t)$ (green line), $\gamma^J_{med}(t)$ (red line) and $\gamma^J_{max}(t)$ (black line). Right panel: distribution of J. Parameter values and initial data as in Fig. S2.

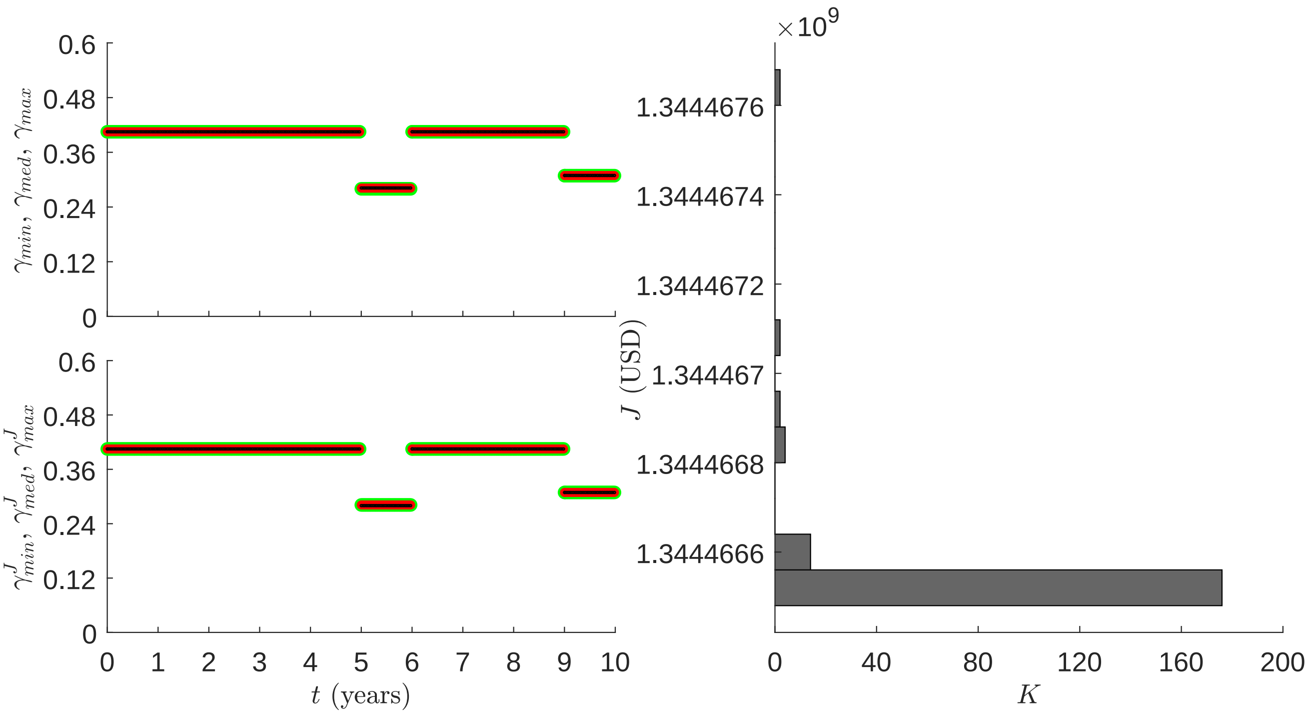

Figure S10: Statistical assessment for the case $C_\gamma = 5C_\gamma^{(o)}$ with 1 year programming and simulation scenario C2. Data obtained by applying $K = 200$ times the PSO algorithm. Left top panel: $\gamma_{min}(t)$ (green line), $\gamma_{med}(t)$ (red line) and $\gamma_{max}(t)$ (black line). Left bottom panel: $\gamma^J_{min}(t)$ (green line), $\gamma^J_{med}(t)$ (red line) and $\gamma^J_{max}(t)$ (black line). Right panel: distribution of J. Parameter values and initial data as in Fig. S1.

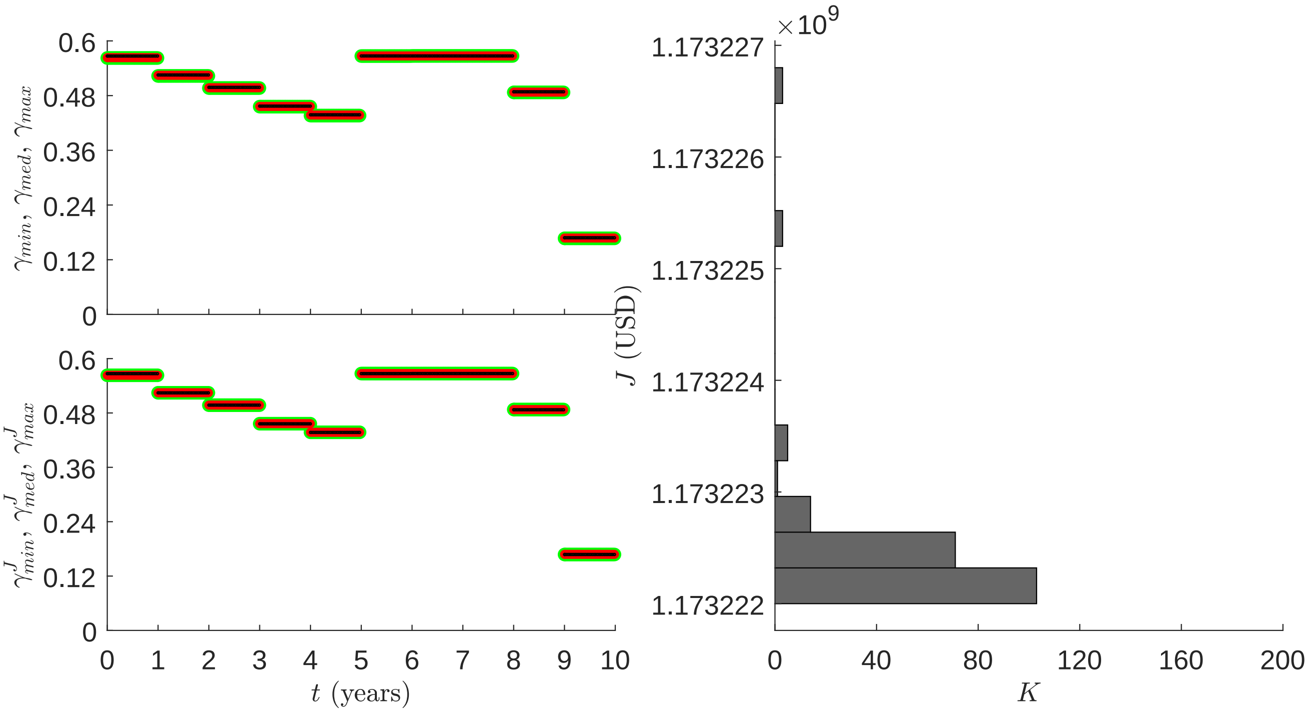

Figure S11: Statistical assessment for the case $C_\gamma = 5C_\gamma^{(o)}$ with 1 year programming and simulation scenario C3. Data obtained by applying $K = 200$ times the PSO algorithm. Left top panel: $\gamma_{min}(t)$ (green line), $\gamma_{med}(t)$ (red line) and $\gamma_{max}(t)$ (black line). Left bottom panel: $\gamma^J_{min}(t)$ (green line), $\gamma^J_{med}(t)$ (red line) and $\gamma^J_{max}(t)$ (black line). Right panel: distribution of J. Parameter values and initial data as in Fig. S1.

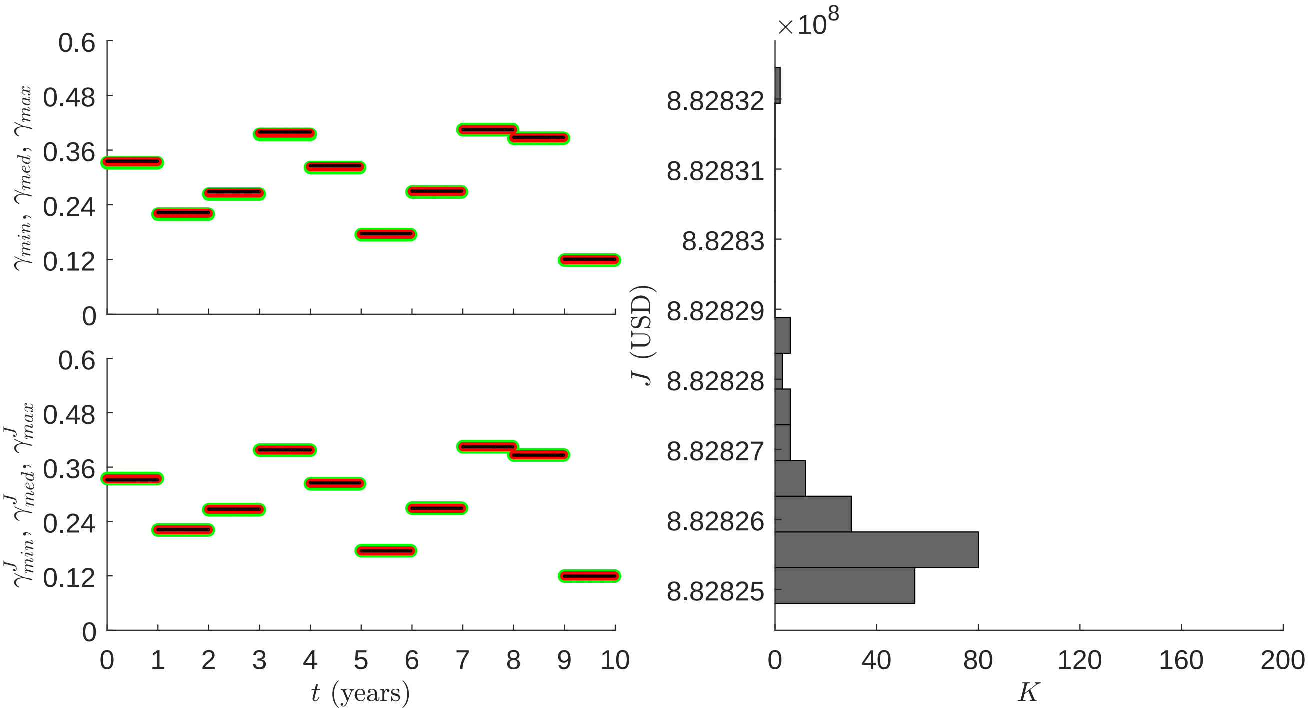

Figure S12: Statistical assessment for the case $C_\gamma = 5C_\gamma^{(o)}$ with 1 year programming and simulation scenario C4. Data obtained by applying $K = 200$ times the PSO algorithm. Left top panel: $\gamma_{min}(t)$ (green line), $\gamma_{med}(t)$ (red line) and $\gamma_{max}(t)$ (black line). Left bottom panel: $\gamma^J_{min}(t)$ (green line), $\gamma^J_{med}(t)$ (red line) and $\gamma^J_{max}(t)$ (black line). Right panel: distribution of J. Parameter values and initial data as in Fig. S1.

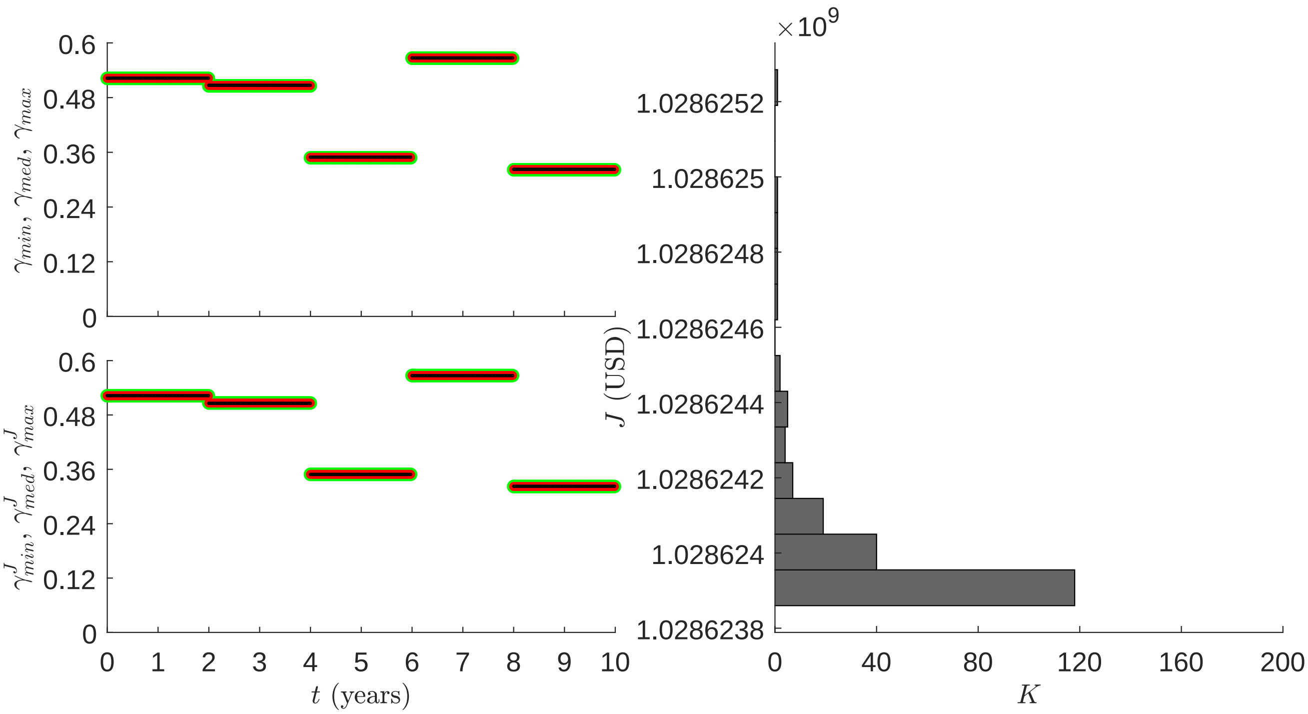

Figure S13: Statistical assessment for the case $C_\gamma = 5C_\gamma^{(o)}$ with 2 years programming and simulation scenario C1. Data obtained by applying $K = 200$ times the PSO algorithm. Left top panel: $\gamma_{min}(t)$ (green line), $\gamma_{med}(t)$ (red line) and $\gamma_{max}(t)$ (black line). Left bottom panel: $\gamma^J_{min}(t)$ (green line), $\gamma^J_{med}(t)$ (red line) and $\gamma^J_{max}(t)$ (black line). Right panel: distribution of J. Parameter values and initial data as in Fig. S2.

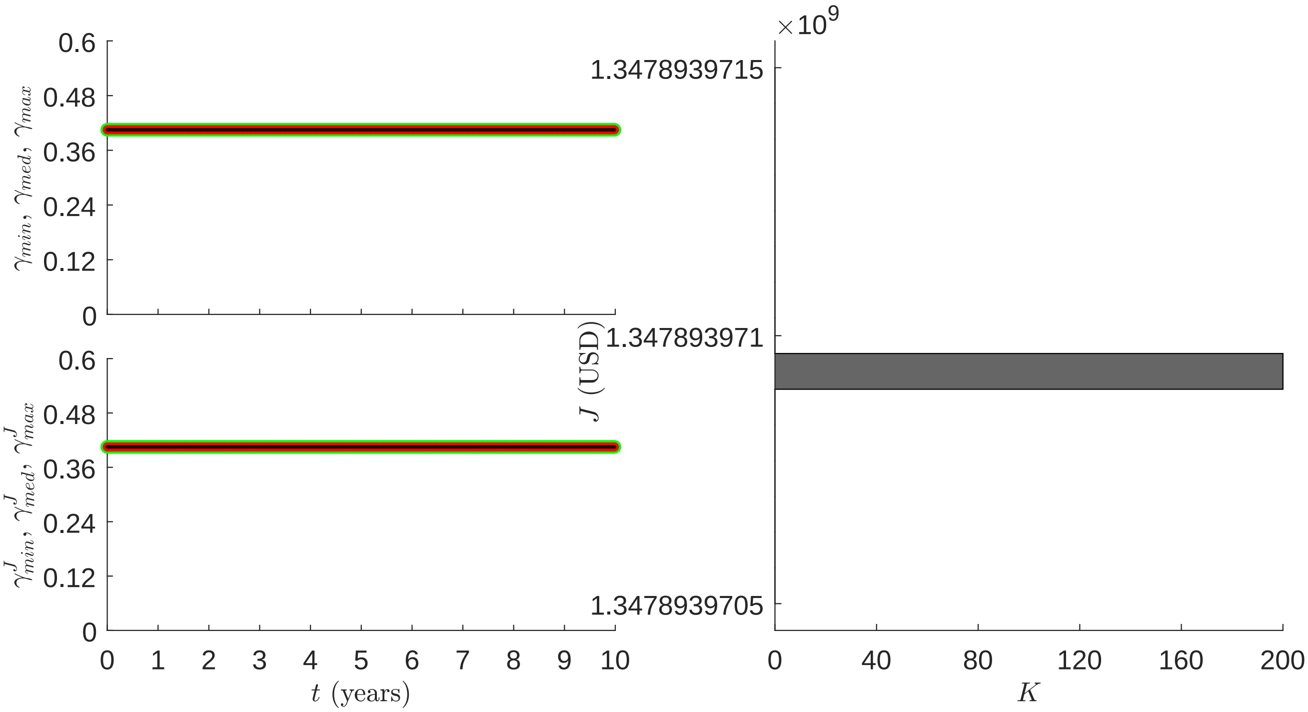

Figure S14: Statistical assessment for the case $C_\gamma = 5C_\gamma^{(o)}$ with 2 years programming and simulation scenario C2. Data obtained by applying $K = 200$ times the PSO algorithm. Left top panel: $\gamma_{min}(t)$ (green line), $\gamma_{med}(t)$ (red line) and $\gamma_{max}(t)$ (black line). Left bottom panel: $\gamma^J_{min}(t)$ (green line), $\gamma^J_{med}(t)$ (red line) and $\gamma^J_{max}(t)$ (black line). Right panel: distribution of J. Parameter values and initial data as in Fig. S1.

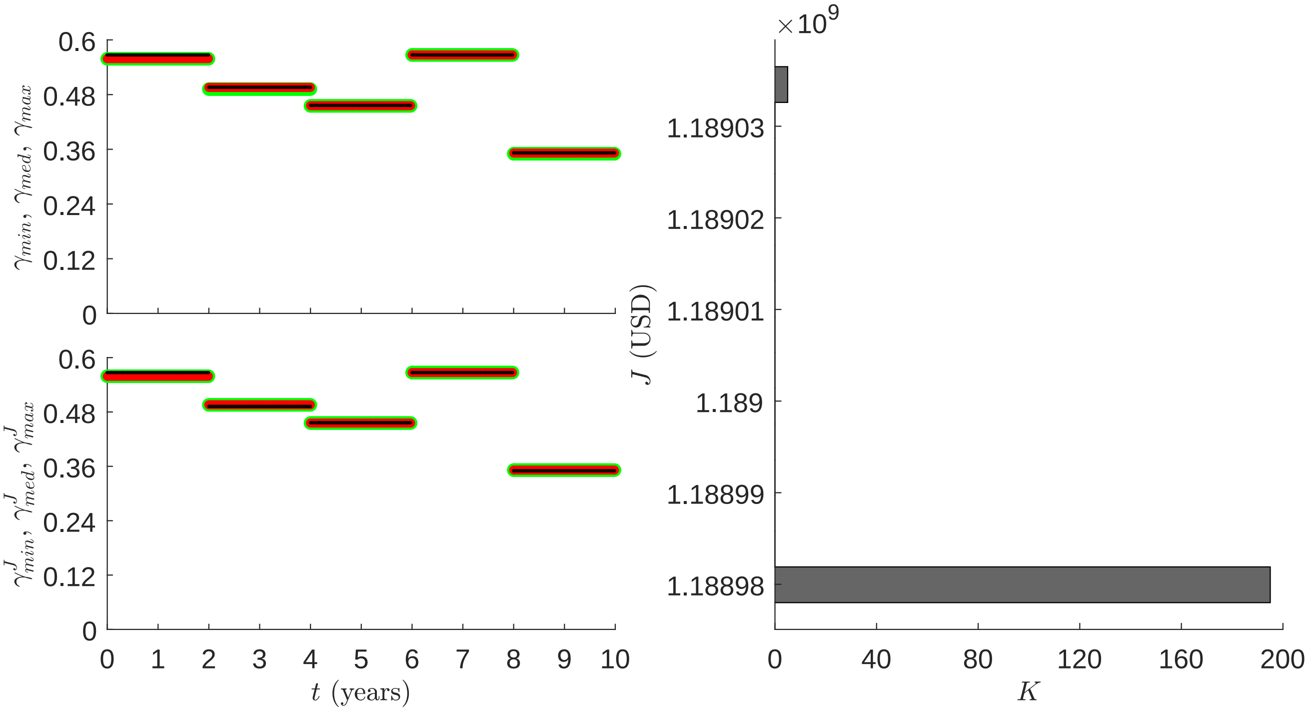

Figure S15: Statistical assessment for the case $C_\gamma = 5C_\gamma^{(o)}$ with 2 years programming and simulation scenario C3. Data obtained by applying $K = 200$ times the PSO algorithm. Left top panel: $\gamma_{min}(t)$ (green line), $\gamma_{med}(t)$ (red line) and $\gamma_{max}(t)$ (black line). Left bottom panel: $\gamma^J_{min}(t)$ (green line), $\gamma^J_{med}(t)$ (red line) and $\gamma^J_{max}(t)$ (black line). Right panel: distribution of J. Parameter values and initial data as in Fig. S1.

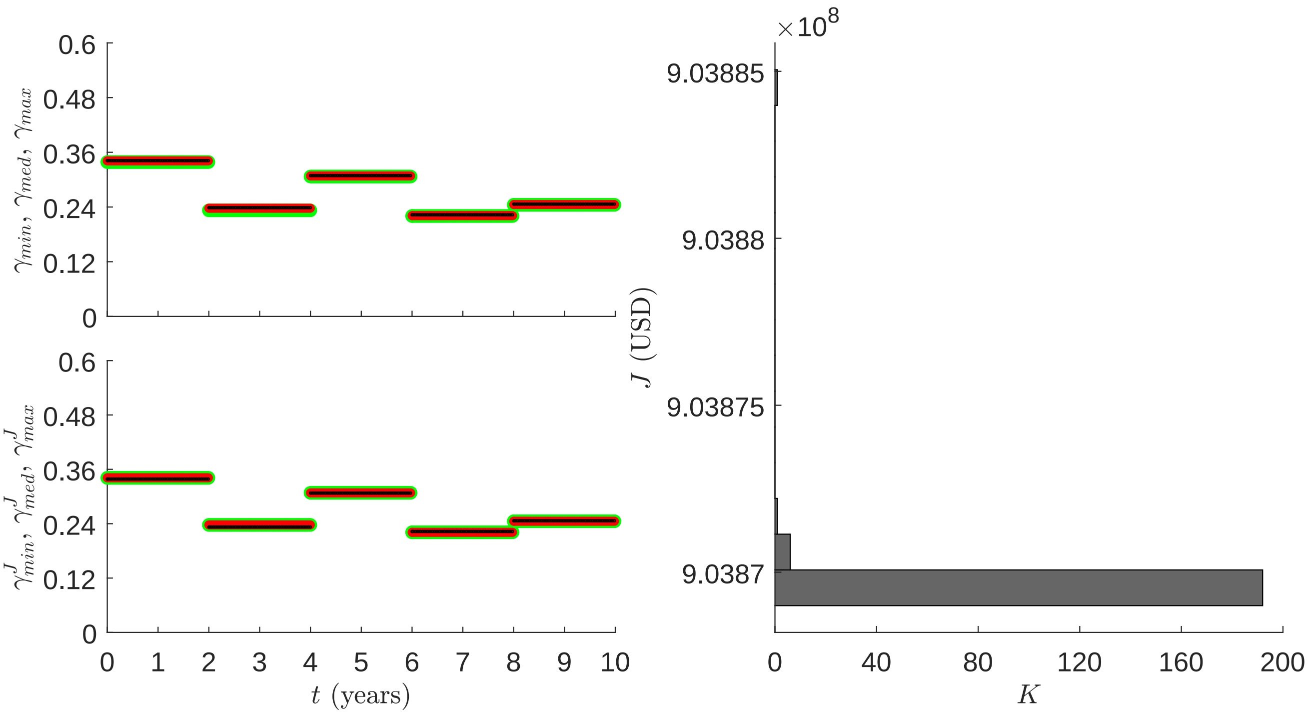

Figure S16: Statistical assessment for the case $C_\gamma = 5C_\gamma^{(o)}$ with 2 years programming and simulation scenario C4. Data obtained by applying $K = 200$ times the PSO algorithm. Left top panel: $\gamma_{min}(t)$ (green line), $\gamma_{med}(t)$ (red line) and $\gamma_{max}(t)$ (black line). Left bottom panel: $\gamma^J_{min}(t)$ (green line), $\gamma^J_{med}(t)$ (red line) and $\gamma^J_{max}(t)$ (black line). Right panel: distribution of J. Parameter values and initial data as in Fig. S1.

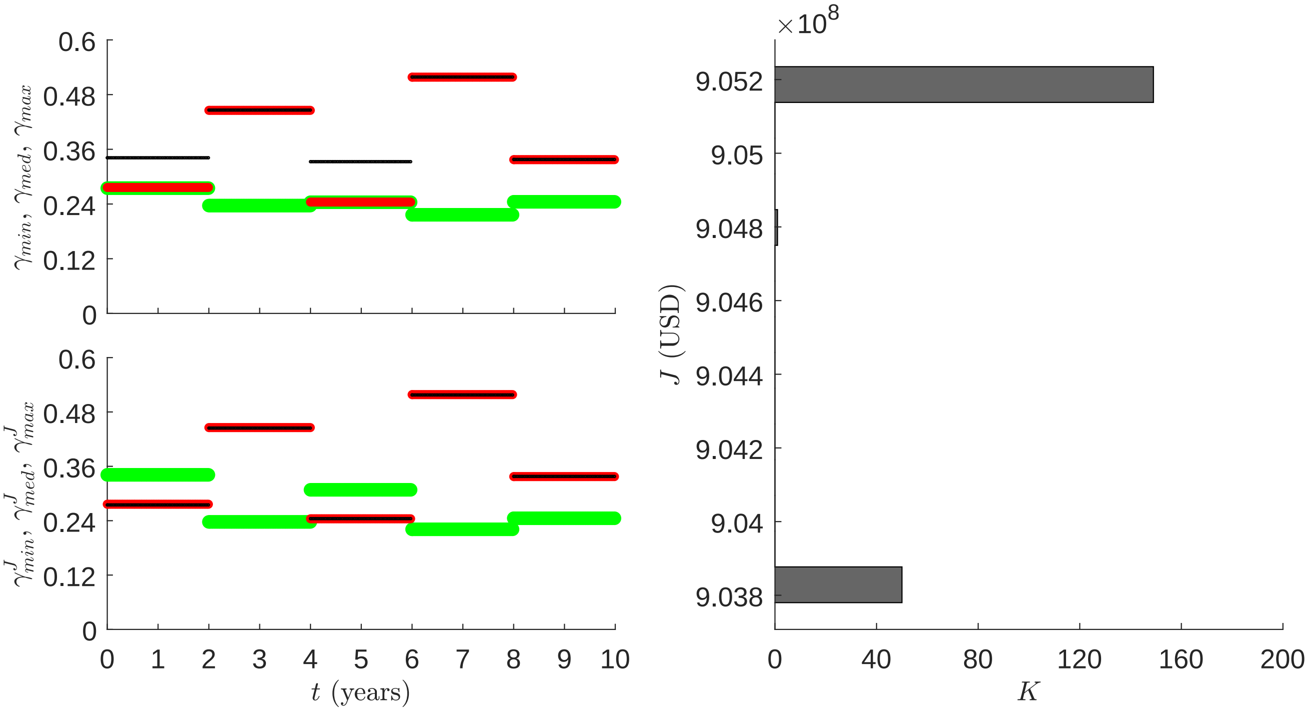

Figure S17: Statistical assessment for the case $C_\gamma = 5C_\gamma^{(o)}$ with 2 years programming and simulation scenario C5. Data obtained by applying $K = 200$ times the PSO algorithm. Left top panel: $\gamma_{min}(t)$ (green line), $\gamma_{med}(t)$ (red line) and $\gamma_{max}(t)$ (black line). Left bottom panel: $\gamma^J_{min}(t)$ (green line), $\gamma^J_{med}(t)$ (red line) and $\gamma^J_{max}(t)$ (black line). Right panel: distribution of J. Parameter values and initial data as in Fig. S2.