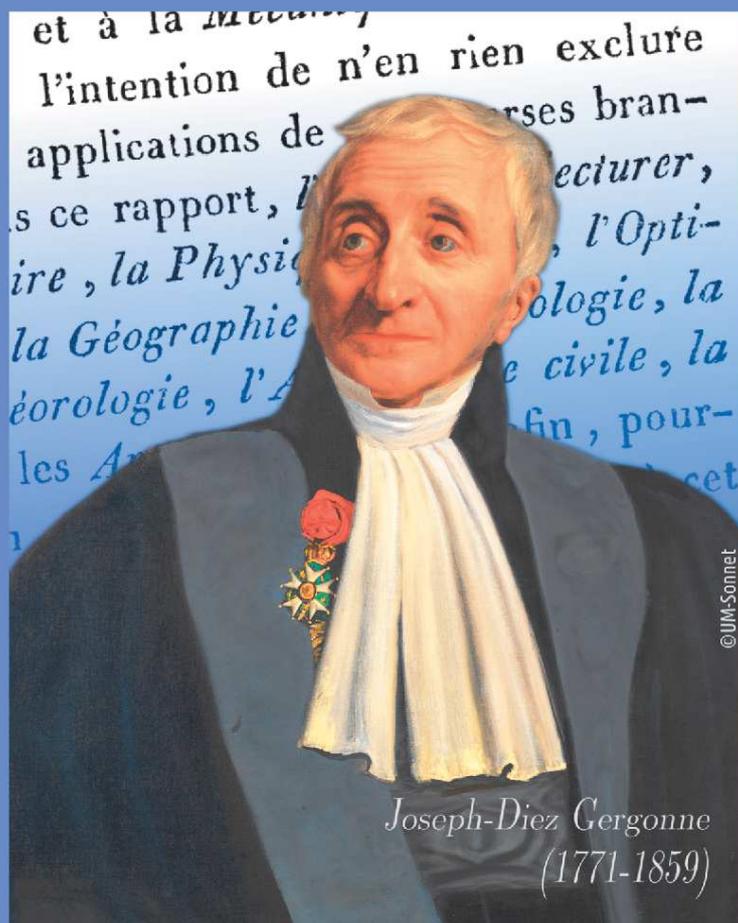


International Study Group  
on the Relations Between the  
**History and Pedagogy  
of Mathematics**

Proceedings of the  
2016 ICME Satellite Meeting



**HPM 2016**

Montpellier, July 18-22, 2016

Edited by:  
Luis Radford  
Fulvia Furinghetti  
Thomas Hausberger



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**Editors:** Luis Radford (Université Laurentienne, Canada)  
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Thomas Hausberger (Université de Montpellier, France)

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# HUMANITIES AND SCIENCES ON MACH'S IDEAS FOR A HIGHER EDUCATION

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## Mach and mathematics education

Mach's position on humanities/sciences dichotomy regarding young people's education is illustrated, stressing his ideas regarding the importance of mathematics teaching.

According to the famous Austrian physicist and philosopher Ernst Mach (the centenary of whose death falls in 2016), studying science – in particular mathematics – proves to be fundamental in helping man to observe and understand the world around him and thus to act in an “economic” way (see, e.g., Mach 1889, pp. 577ff.); with this viewpoint, scientific education should be strongly pursued. There are in fact illuminating examples of application of Mach's ideas in mathematics education of his time (see Zuccheri & Zudini 2008).

## The formative value of mathematics and sciences

*Über den relativen Bildungswert der philologischen und der mathematisch-naturwissenschaftlichen Unterrichtsfächer der höheren Schulen* is the significant title of a conference held in 1886 by Mach and contained in his *Populär-wissenschaftliche Vorlesungen* (Mach 1896, pp. 338-374). Mach shows himself to be a very modern scholar in his treatment of the relationship between humanities and sciences and their formative value. He argues that, within the cultural development of his time, humanities cannot be considered any longer to be the only (nor even the better) means to offer a higher education.

Mach counters the usual arguments in favor of the supremacy of humanistic culture with the greater value and effectiveness of teaching mathematics and science (which for certain formative aspects cannot be separated). He gives specific examples, claiming their superiority with regard to educational aims and for the development of ability in observation and logic (see Mach 1896, pp. 344ff.). He provides a series of directions to be implemented for an improvement of mathematics and science education (see Mach, 1896, pp. 364ff.).

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