Transient model of a Professional Oven

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Abstract

Tackling the climate change by reducing energy consumption is among the biggest, most urgent challenges society is facing and requires a continuous efficiency improvement of thermal systems. Appropriate design strategies, developed a priori and then experimentally validated according to suitable test protocols on a prototype, are needed in order to reach potential energy saving targets. These strategies can successfully be implemented in the food service sector, where cooking appliances, in particular, present many possibilities for improving energy savings. Therefore, a valuable design methodology should take into account not only steady state operating conditions but also the transient behaviours of the device, which must be described by means of specially developed theoretical dynamic models. The operating profile of an oven, for example, consists of a sequence of unsteady phases (cavity heating-up, food introduction and extraction, switching from one cooking mode to another) interspersed with steady cooking phases. The dynamic model presented in this paper defines the energy conservation equations of a professional oven, where a high temperature thermal source positioned inside its cavity produces thermal power radiated and modulated over time, according to a suitable control strategy. In particular, when the temperature in the cooking zone of the cavity has reached a specified set point, this is thermostatically controlled in time, depending on the cooking phase. The resulting equation system is then solved by means of numerical methods. With this code, it is possible to support the design phase of both the structure and the control strategy of the oven. It permits, for example, to get a general understanding of the best possible configurations and combinations of insulation materials for the cavity walls or, with reference to the control strategy, to simulate different cooking procedures, with the aim of optimizing the operating sequence of the oven, reaching the maximum energy saving without reducing the cooking quality. The code, validated by comparison with a set of experimental data obtained with a current production model, will be applied in the design phase of a new line of high efficiency professional ovens.

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1. Introduction

The current global environmental situation requires technical efforts for designing new systems and machines capable to minimize the overall energy consumption. Professional kitchens, having an average energy demand almost twice the amount used for domestic cooking, has a substantial energy saving potential. To meet the increasing demand for more sustainable kitchen products, it is necessary to develop suitable design strategies, based on theoretical models experimentally validated according to rigorous and significant test protocols on a prototype.

The models should take into account not only steady state operating conditions, but also the transient behaviours of the device, which must be described with specially developed theoretical dynamic models. In fact, transitory situations are very common and time consuming during the operating life of a professional cooking appliance: the operating profile of an oven, for example, consists of a sequence of unsteady phases (cavity heating-up, food introduction and extraction, switching from one cooking mode to another) interspersed with steady cooking steps.

To characterize the energy performances of the system it is necessary to identify and quantify the incoming and outgoing energy flows from the device, including the evaluation of internal and external heat transfers during both the transitory and the steady state phases. The scope of such an analysis is not only to do an energetic performance evaluation of a particular appliance but, more generally, to give support to the design phase, the material selection, the cooking phases evaluation and the control logic definition from the beginning.

The aim of this paper is the definition and validation of a model of a professional oven. Several studies dealing with the modelling of an oven can be found in the open literature: they can be organized according to five possible approaches, i.e. three or two dimensional CFD models, system modelling for control purposes, algebraic modelling and applications of the lumped capacitance method, as briefly discussed in the following review.

The most accurate and detailed 3D-CFD approach is adopted in [1], [2], [3]. In [1] authors present a model of an electric bread-baking oven, comparing the internal temperature profiles obtained with three radiation models.

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Greek symbols

δᵢⱼ  Kronecker Delta
Δx   spatial step [m]
In [2] the authors evaluate heat transfer and fluid flows inside a domestic gas oven. Reference [3] presents the model of a pilot scale convection oven, used to study the reduction of the energy consuming pre-heating time. All these models are validated by comparison with experimental data with fairly limited discrepancy, also at different thermal levels and geometries of the internal baffle plate [3].

Other authors have analysed the thermal behaviour of an oven by means of 2D-CFD numerical models. In [4], an electric static oven, used for bread baking, is analysed to calculate the heat exchanged with the product. Authors present a comprehensive methodology for evaluating the heat fluxes around the test material, considering natural convection, infrared radiation and conduction through a cement slab. Heat equations are solved on a cross section of the cavity, for all components of the oven, with the finite element method, using a parametric solver. Numerical results are in good agreement with heat flux measurements on the upper surface of a polymeric cylindrical sample. The scope of the model was to test the influence on energy consumption of different operating conditions, obtained lowering the cooking temperature, increasing the incident radiative heat flux and lowering the thermal capacity of the oven.

Another approach, used in the designing of control systems, considers simple thermal models, able to describe the temperature dynamics of an oven cavity. In [5], the authors present a set of mathematical models, which relate the input power and the air temperature inside a forced convection oven, based on experimentally determined transfer functions.

A modelling approach based on simple algebraic models is presented in [6]. The authors predict the heat transfer to a load positioned in an electric oven, considering the contributions of natural convection and radiation. To allow an analytical solution of the model equation, the radiative transfer term was linearized considering the temperature differences between the oven walls and the surface of the thermal load, instead of being driven by the fourth-power. The analysis takes into account changes of size, shape, materials, radiation surface properties and oven set point temperatures, showing discrepancies of about 1% between predicted and experimental data.

Another approach, based on lumped capacitance method, is presented in [7]. The authors state that such an approach, used for building energy simulation [8] with apparently good results, have never been used to model ovens before. It simplifies the heat transfer equations by considering the system as a discrete set of thermal capacitances and resistances, permitting to have good results in transient heating and cooling problems [7], [9].

In our investigation, we have developed a lumped capacitance model of a professional oven, characterized by the interactions between two thermal zones, i.e. the power zone and the cooking zone (Fig. 1.b), and by the adoption of a not linearized radiation heat transfer approach. As a whole, the main features of this model are:

- the analysis of the energy exchange between the two zones of the oven;
- the modelling of external energy exchange, taking into account the temperature outside the glass door;
- it considers all the heat exchange mechanisms (radiation, forced convection due to the fan and conduction);
- the number of nodes inside the material is customizable, to evaluate the impact of layers of different insulation materials on the oven efficiency;
- it can be easily modified for evaluating new designs with low computational resources.

2. The oven and its model

2.1. The oven

The oven used for the analysis is an Electrolux AoS Touchline 10 GN 1/1 lengthwise. It has an internal cavity with a volume of 0.335 m³ and a capacity of 10 GN 1/1 trays. It can operate in three modalities: forced convection, steam, combined convection and steam. The declared power (for power supply at 400V, AC 3N 50Hz) is 17 kW in forced convection and in steam cooking modes (using the resistors in the boiler) and other 0.5 kW absorbed by auxiliaries [10]. The oven cavity is composed of two adjoining sections: the rear part, or power zone, occupied by the fan and the heat exchanger elements, and the cooking zone (Figure 1.b). The suction wall, that separates the two zones, has two functions: guiding the air flow to the fan and directing the heated air into the cooking zone [10]. The power zone has the following dimensions: height = 0.76 m, width = 0.65 m, length = 0.2 m, while the cooking zone has height = 0.76 m, width = 0.65 m and length = 0.48 m.
2.2. The thermal model

Aim of the model is the evaluation of the energy performance of the oven during transient and steady state operations. The numerical model considers two zones, which contain only dry air and oven accessories (fan, trays, etc.); these have a high impact on the thermal inertia of the system. The presence of the food is not considered. All the components are modeled with the lumped capacitance method, so they interact as lumped elements of an electric circuit: the potential nodes refer to the temperature of each element, a set of capacitors reproduce the thermal capacity while electric resistances are used to indicate the convective, radiative or conductive thermal interaction between the elements. The model considers the two zones of the professional oven separately, as previously described: the power zone (hereinafter indicated with the subscript \( P \)) and the cooking zone (hereinafter indicated with the subscript \( C \)). Spatial discretization is considered for the walls (hereinafter indicated with the subscript \( w_i \) for the \( i \)-th wall), divided in one dimensional isothermal layers. The layers are numbered starting from the external one (indicated with the subscript \( w_1 \)) to the internal superficial one (indicated with the subscript \( w_N \)), see Fig. 3a. The total number of walls in each zone is indicated with \( N \).

2.2.1. The power zone

The logical scheme of the power zone is shown in Fig. 2. This zone is bounded by five continuous walls and interfaced with the cooking zone through the suction wall, Fig. 1.b.

![Functional scheme of the power zone of the oven.](image)

The power zone is characterized by the presence of the heaters (electric resistor); they are modeled with two nodes (one internal and one external) due to the resistor high thermal inertia. The thermal power, \( P_{el} \), is generated only inside the volume of the inner node. The external node interacts both with the air through convection and with the walls through radiative heat transfer. The presence of the fan and other accessories is considered in the thermal...
mass node (indicated with the subscript \(m_P\)). The fan is responsible of the enthalpy flux exchange between the power and the cooking zones, \(m_{fan}c_p(T_p - T_c)\). The corresponding energy balance equations for every node are given below (Eqs.1-5).

\[
V_P p_P c_p \frac{T_p n - T_p n-1}{t_n - t_{n-1}} = \sum_i h_{conv} w_{j,i} A_{w,j}(T_{w,j,n} - T_{p,n}) + h_{conv} m_P A_{mP}(T_{mP,n} - T_{p,n}) + h_{conv} R_1 A_{R1}(T_{R1,n} - T_{p,n}) + m_{fan}c_p(T_{C,n} - T_{p,n})
\]

\[
V_{R1} p_{R1} c_{R1} \frac{T_{R1,n} - T_{R1,n-1}}{t_n - t_{n-1}} = \frac{A_{R1}}{R_{cond} R_1}(T_{R2,n} - T_{R1,n}) + h_{conv} R_1 A_{R1}(T_{p,n} - T_{R1,n}) + \sum_j P_{rad} w_{j,n}
\]

\[
V_{R2} p_{R2} c_{R2} \frac{T_{R2,n} - T_{R2,n-1}}{t_n - t_{n-1}} = \frac{A_{R1}}{R_{cond} R_1}(T_{R1,n} - T_{R2,n}) + P_{el,n}
\]

\[
V_{mP} p_{mP} c_{mP} \frac{T_{mP,n} - T_{mP,n-1}}{t_n - t_{n-1}} = h_{conv} m_P A_{mP}(T_{p,n} - T_{mP,n})
\]

\[
V_{w,j} p_{w,j} c_{w,j} \frac{T_{w,j,n} - T_{w,j,n-1}}{t_n - t_{n-1}} = h_{conv} w_{j} A_{w,j}(T_{p,n} - T_{w,j,n}) + \frac{\lambda}{\Delta x} A_{w,j}(T_{w,j,n-1} - T_{w,j,n}) + \sum_j (1 - \delta_{ij}) P_{rad} w_{j,n}
\]

In Eq.5, the subscript \(j\) indicates a wall different from the \(i\)th one.

2.2.2. The cooking zone

The logical scheme of the cooking zone is shown in Fig.3b. This zone has the same number of bounding walls of the power zone, but it is characterized by the presence of the door, which is composed by two glasses with an air gap between them. The door is modeled with two nodes: the internal node (subscript \(g1\)) and the outer node (subscript \(g2\)). The internal node interacts with the cooking zone by convection (the radiation with the walls is neglected since the door is made with a low-emission glass) and with the outer node by means of an overall thermal resistance, indicated in the figure as \(R_g\). Furthermore, the area of inner glass is equal to the area of the outer glass, \(A_{g1} = A_{g2}\).

The corresponding energy balance equations for every node are given below (Eqs.6-12).

\[
V_C p_C c_C \frac{T_{C,n} - T_{C,n-1}}{t_n - t_{n-1}} = \sum_i h_{conv} w_{j,i} A_{w,j}(T_{w,j,n} - T_{C,n}) + h_{conv} m_C A_{mC}(T_{mC,n} - T_{C,n}) + m_{fan}c_C(T_{P,n} - T_{C,n})
\]

\[
V_{mC} p_{mC} c_{mC} \frac{T_{mC,n} - T_{mC,n-1}}{t_n - t_{n-1}} = h_{conv} m_C A_{mC}(T_{C,n} - T_{mC,n})
\]

\[
V_{w,j} p_{w,j} c_{w,j} \frac{T_{w,j,n} - T_{w,j,n-1}}{t_n - t_{n-1}} = h_{conv} w_{j} A_{w,j}(T_{C,n} - T_{w,j,n}) + \frac{\lambda}{\Delta x} A_{w,j}(T_{w,j,n-1} - T_{w,j,n}) + \sum_j (1 - \delta_{ij}) P_{rad} w_{j,n}
\]

\[
V_{w,j-1} p_{w,j} c_{w,j-1} \frac{T_{w,j-1,n} - T_{w,j-1,n-1}}{t_n - t_{n-1}} = \frac{\lambda}{\Delta x} A_{w,j}(T_{w,j-2,n} - T_{w,j-1,n}) + \frac{\lambda}{\Delta x} A_{w,j}(T_{w,j,n} - T_{w,j-1,n})
\]

\[
V_{w,j} p_{w,j} c_{w,j} \frac{T_{w,j,n} - T_{w,j,n-1}}{t_n - t_{n-1}} = h_{conv} e A_{w,j}(T_e - T_{w,j,n}) + \frac{\lambda}{\Delta x} A_{w,j}(T_{w,j+1,n} - T_{w,j,n})
\]

Figure 3. a) Functional scheme of a composite wall; b) functional scheme of the cooking zone of the oven.
The fan is responsible of the enthalpy flux exchange between the power and the cooking zones.

The corresponding energy balance equations for every node are given below (Eqs. 1-5).

\[
V_{g1} \rho_{g1} C_{g1} \frac{T_{g1,n} - T_{g1,n-1}}{t_{n-1}} = h_{conv g1} A_{g1} \left( T_{c,n} - T_{g1,n} \right) + \frac{A_{g1}}{R_g} \left( T_{g2,n} - T_{g1,n} \right) \\
V_{g2} \rho_{g2} C_{g2} \frac{T_{g2,n} - T_{g2,n-1}}{t_{n-1}} = h_{conv e} A_{g2} \left( T_{e} - T_{g2,n} \right) + \frac{A_{g2}}{R_g} \left( T_{g1,n} - T_{g2,n} \right)
\]

\[\text{(1)}\]

\[\text{(2)}\]

\[\text{(3)}\]

\[\text{(4)}\]

\[\text{(5)}\]

The subscript \(j\) indicates a wall different from the \(i\)th one.

2.2.2. The cooking zone

The logical scheme of the cooking zone is shown in Fig. 3b. This zone has the same number of bounding walls of the power zone, but it is characterized by the presence of the door, which is composed by two glasses with an air gap between them. The door is modeled with two nodes: the internal node (subscript \(i\)) and the outer node (subscript \(o\)). The internal node interacts with the cooking zone by convection (the radiation with the walls is neglected since the door is made with a low-emission glass) and with the outer node by means of an overall thermal resistance, indicated in the figure as \(R\). Furthermore, the area of inner glass is equal to the area of the outer glass.

The corresponding energy balance equations for every node are given below (Eqs. 6-12).

\[\text{(6)}\]

\[\text{(7)}\]

\[\text{(8)}\]

\[\text{(9)}\]

\[\text{(10)}\]

\[\text{(11)}\]

\[\text{(12)}\]

2.2.3. Radiation heat transfer

Radiation has a big impact in the heat transfer process in a professional oven, in particular in the power zone where the resistor radiates with high temperature difference towards the walls and the other elements. In building energy simulation, but also in the case of ovens analysis [6], radiation is usually modeled with the linearization theory. With such an approach, a coefficient of radiative heat transfer multiplies the temperature difference between two surfaces, so that the radiation term becomes linear, but some assumptions are required to estimate the coefficient itself, which depends on several factors (geometry, temperatures, etc.). In building simulation, the geometry is simple and temperature differences are quite low thus, the linear model is sufficient to have a good evaluation of radiative heat transfer. In the present case, due to both the complex geometry of the oven and the high temperatures, the linear model is not taken into account. The radiation heat transfer is here modeled directly with an explicit scheme, without linearization and with the evaluation of the view factors between the resistor and the walls. For the last purpose, it has been used a MATLAB© script developed by [11]: it is a function that uses CDIF (Contour Double Integral Formula) to calculate view factors between planar surfaces (polygons). Consequently, the main assumption of the model is that the resistor is seen by each surface as a plane surface. The heat transmitted by radiation is thus calculated with the approach called the \textit{net radiation method for encloses}. A formal explanation of this theory is presented in [12] and correspond to Eq. 13:

\[
\sum_{j=1}^{N} \left( \frac{\delta_{ij}}{\epsilon_j} - F_{i-j} \frac{1 - \epsilon_j}{\epsilon_j} \right) \sigma \left( T_{j,n}^4 - T_{n-1}^4 \right) = \sum_{i=1}^{N} (\delta_{ij} - F_{i-j}) \sigma (T_{i,n-1}^4 - T_{i,n}^4)
\]

where \(i\) has the value \(1,2,...,N\) for each surface. Temperatures at the time step \(n-1\) are used to calculate the radiative heat exchanged at the time step \(n\) (explicit scheme). Air is considered transparent to radiation.

2.3. Tuning procedure

The following four parameters are selected as tuning parameters for setting up the model:

- the averaged convective coefficient between air and walls: \(h_{conv ws}\);
- the averaged convective coefficient between air and thermal resistance, multiplied by resistor surface: \(h_{conv R1}A_{R1}\);
- the averaged convective coefficient between air and thermal masses, multiplied by the interface area: \(h_{conv MC}A_{MC}, h_{conv MP}A_{MP}\).

A tuning procedure based on the trial and error method has been used for setting up the four parameters. A comparison between numerical and experimental data was made analysing the slope of the transitory state (initial heating up phase), see Fig. 4 and of the ramp up and ramp down in steady state phases, see Fig. 5. In order to check the physical meaning of the parameters, for two of them \((h_{conv ws} and h_{conv R1}A_{R1})\) it was possible to compare the results with those obtained with theoretical correlations. The average convective coefficient between air and walls is calculated through the averaged Nusselt number obtained with the heat transfer correlation, valid for isothermal flat plates, defined by Eq. 14 [13]:

\[
\overline{Nu} = \frac{h_{conv ws} L}{\lambda} = 0.037 \ Re_{L}^{0.7} Pr^{1/3} \quad 0.6 < Pr < 60 \quad 5 \times 10^5 < Re_L < 10^7
\]

\[\text{(13)}\]

\[\text{(14)}\]

\(Re_L\) is calculated with a velocity of 10 m/s, obtained from known CFD values, and a characteristic length of the walls \(L = 0.7\ m\). The averaged convective coefficient between air and heaters was compared with a heat transfer correlation used for the crossflow across tube banks [13]. The averaged Nusselt number is given by Eq. 15:
where the coefficients, which depend on the geometry of the tubes, have been assumed equal to $C_1 = 0.76$, $C = 0.4$, $m = 0.6$ [13]. Also in this case the mean velocity, $15 \text{ m/s}$, is a result of known CFD analysis for this type of professional oven, while the heaters diameter $D = 0.006 \text{ m}$ is considered as characteristic length. For the other two parameters ($h_{\text{conv mc}}A_{mc}$ and $h_{\text{conv mp}}A_{mp}$) no theoretical considerations can be made. The following Tab. 1 presents the values of the parameters obtained with the tuning procedure.

The correlation values are different from the tuning ones: in fact the heat transfer correlations are valid for heaters tubes in perfect crossflow and fluid flow parallel to the surface, while in the oven these conditions are only partly satisfied. Moreover the differences are due also to the use of the averaged value of velocity.

### 3. Results and discussion

The control logics used in the model is on/off for the resistor power, with a fixed set point, and a deadband for the activations of the heating power, based on the power zone temperature. With the chosen values of the tuning parameters the model shows a good agreement with the experimental data as shown in the following Figs. 4 and 5.

With the present model, it is therefore possible to face the challenging task of studying the thermodynamic of a professional oven, which has to ensure high productivity with high quality.

Advanced control strategies can also be developed, to increase the performance of the device according to several parameters, i.e. the monitored temperature of a sample inside the cavity, or the energy exchanged with it. Figs. 6 and 7 present some preliminary numerical results. In the first, heaters temperature and activation times are reported together with the air temperatures in the two oven zones. In the second, the radiation heat transfer exchanged by the heaters is shown: it is clear how radiation influences the transient state, while during steady state it has a lower impact, since the temperature difference between the heaters and the surrounding components is lower.
4. Conclusions and future works

In this article, a numerical model of a professional oven has been presented. This model is based on the lumped capacitance method, that has demonstrated the capability of predicting the thermodynamic performances of the oven with low computational efforts compared to other numerical techniques (i.e. CFD). The oven presents two distinct zones: the power zone and the cooking cavity. The model is capable of predicting the thermodynamic behavior of both the zones and it permits to evaluate the overall energy performances of the device. The oven works mainly in forced convection so it was required an estimation of the convection parameters with a tuning procedure to find their averaged value. The contribution of radiation heat transfer has also been modeled with a quite accurate approach based on the so-called net radiation method for encloses.

The results have been compared with a set of experimental data showing a good agreement in both the transitory and the stationary operating phases. The last part of the article presents the capabilities of the model to predict in detail the thermodynamic performances of the oven in given operating conditions. Future work will focus on the following activities:

- comparison with more experimental data at different cooking modes;
- an optimization study to find correlations of the tuning coefficients according to different operating conditions;
- introduction of the hygrometric balance in a combined oven (presence of steam produced by a dedicated boiler);
- simulation of the food thermodynamic behavior inside the oven.

References