Constraints on the chiral magnetic effect using charge-dependent azimuthal correlations in \( pPb \) and \( PbPb \) collisions at the CERN Large Hadron Collider

A. M. Sirunyan et al.*
(CMS Collaboration)

(Received 4 August 2017; published 23 April 2018)

Charge-dependent azimuthal correlations of same- and opposite-sign pairs with respect to the second- and third-order event planes have been measured in \( pPb \) collisions at \( \sqrt{s_{\text{NN}}} = 8.16 \) TeV and \( PbPb \) collisions at 5.02 TeV with the CMS experiment at the LHC. The measurement is motivated by the search for the charge separation phenomenon predicted by the chiral magnetic effect (CME) in heavy ion collisions. Three- and two-particle azimuthal correlators are extracted as functions of the pseudorapidity difference, the transverse momentum \((p_T)\) difference, and the \( p_T \) average of same- and opposite-charge pairs in various event multiplicity ranges. The data suggest that the charge-dependent three-particle correlators with respect to the second- and third-order event planes share a common origin, predominantly arising from charge-dependent two-particle azimuthal correlations coupled with an anisotropic flow. The CME is expected to lead to a \( v_2 \)-independent three-particle correlation when the magnetic field is fixed. Using an event shape engineering technique, upper limits on the \( v_2 \)-independent fraction of the three-particle correlator are estimated to be 13\% for \( pPb \) and 7\% for \( PbPb \) collisions at 95\% confidence level. The results of this analysis, both the dominance of two-particle correlations as a source of the three-particle results and the similarities seen between \( PbPb \) and \( pPb \), provide stringent constraints on the origin of charge-dependent three-particle azimuthal correlations and challenge their interpretation as arising from a chiral magnetic effect in heavy ion collisions.

DOI: 10.1103/PhysRevC.97.044912

I. INTRODUCTION

It has been suggested that in high-energy nucleus-nucleus (AA) collisions, metastable domains of gluon fields with nontrivial topological configurations may form [1–4]. These domains can carry an imbalance between left- and right-handed quarks arising from interactions of chiral quarks with topological gluon fields, leading to a local parity \((P)\) violation [3,4]. This chirality imbalance, in the presence of the extremely strong magnetic field, which can be produced in a noncentral AA collision, is expected to lead to an electric current perpendicular to the reaction plane, resulting in a final-state charge separation phenomenon known as the chiral magnetic effect (CME) [5–7]. Such macroscopic phenomena arising from quantum anomalies are a subject of interest for a wide range of physics communities. The chiral-anomaly-induced phenomena have been observed in magnetized relativistic matter in three-dimensional Dirac and Weyl materials [8–10]. The search for the charge separation from the CME in AA collisions was first carried out at RHIC at BNL [11–15] and later at the CERN LHC [16] at various center-of-mass energies. In these measurements, a charge-dependent azimuthal correlation with respect to the reaction plane was observed, which is qualitatively consistent with the expectation of charge separation from the CME. No strong collision energy dependence of the signal is observed going from RHIC to LHC energies, although some theoretical predictions suggested that the possible CME signal could be much smaller at the LHC than at RHIC because of a shorter lifetime of the magnetic field [17]. Nevertheless, theoretical estimates of the time evolution of the magnetic field have large uncertainties [17].

The experimental evidence for the CME in heavy ion collisions remains inconclusive because of several identified sources of background correlations that can account for part or all of the observed charge-dependent azimuthal correlations [18–20]. Moreover, the charge-dependent azimuthal correlation in high-multiplicity \( pPb \) collisions has been recently found to have a nearly identical value to that observed in \( PbPb \) collisions [21]. This is a strong indication that the observed effect in heavy ion collisions might predominantly result from background contributions. The CME-induced charge separation effect is predicted to be negligible in \( pPb \) collisions, as the angle between the magnetic field direction and the event plane is expected to be randomly distributed [21,22].

The charge separation can be characterized by the first \( P \)-odd sine term \((a_1)\) in a Fourier decomposition of the charged-particle azimuthal distribution [23]:

\[
\frac{dN}{d\phi} \propto 1 + 2 \sum_n \left[ v_n \cos[n(\phi - \Psi_{\text{RP}})] + a_n \sin[n(\phi - \Psi_{\text{RP}})] \right],
\]

where \( \phi - \Psi_{\text{RP}} \) represents the particle azimuthal angle with respect to the reaction plane angle \( \Psi_{\text{RP}} \) in heavy ion collisions.
(determined by the impact parameter and beam axis), and \( v_n \) and \( a_n \) denote the coefficients of \( P \)-even and \( P \)-odd Fourier terms, respectively. Although the reaction plane is not an experimental observable, it can be approximated in heavy ion collisions by the second-order event plane \( \Psi_2 \), determined by the direction of the beam and the maximal particle density in the elliptic azimuthal anisotropy. The \( P \)-odd terms will vanish after averaging over events, because the sign of the chirality imbalance changes event by event. Therefore, the observation of such an effect is only possible through the measurement of particle azimuthal correlations. An azimuthal three-particle correlator \( \gamma_{112} \) proposed to explore the first coefficient \( a_1 \) of the \( P \)-odd Fourier terms characterizing the charge separation [23] is

\[
\gamma_{112} \equiv \langle \cos(\phi_\alpha + \phi_\beta - 2\Psi_2) \rangle = \langle \cos(\phi_\alpha - \Psi_2) \cos(\phi_\beta - \Psi_2) \rangle - \langle \sin(\phi_\alpha - \Psi_2) \sin(\phi_\beta - \Psi_2) \rangle. \tag{2}
\]

Here, \( \alpha \) and \( \beta \) denote particles with the same or opposite electric charge sign and the angle brackets reflect an averaging over particles and events. Assuming particles \( \alpha \) and \( \beta \) are uncorrelated, except for their individual correlations with respect to the event plane, the first term on the right-hand side of Eq. (2) becomes \( \langle v_{1,\alpha} v_{1,\beta} \rangle \), which is generally small and independent of the charge [12], while the second term is sensitive to the charge separation and can be expressed as \( \langle a_{1,\alpha} a_{1,\beta} \rangle \).

While the similarity of the \( pPb \) and \( PbPb \) data at 5.02 TeV analyzed by the CMS experiment pose a considerable challenge to the CME interpretation of the charge-dependent azimuthal correlations observed in AA collisions [21], important questions still remain to be addressed: is the correlation signal observed in \( pPb \) collisions entirely a consequence of background correlations? What is the underlying mechanism for those background correlations that are almost identical in \( pPb \) and \( PbPb \) collisions? Can the background contribution be quantitatively constrained with data and, if so, is there still evidence for a statistically significant CME signal?

In particular, among the proposed mechanisms for background correlations, one source is related to the charge-dependent two-particle correlation from local charge conservation in decays of resonances or clusters (e.g., jets) [20]. By coupling with the anisotropic particle emission, an effect resembling charge separation with respect to the reaction plane can be generated. The observed characteristic range of the two-particle correlation in data is around one unit of rapidity, consistent with short-range cluster decays. In this mechanism of local charge conservation coupled with the elliptic flow, a background contribution to the three-particle correlator, \( \gamma_{112} \), is expected to be [24]

\[
\gamma_{112}^{bkg} = k_2 \langle \cos(\phi_\alpha - \phi_\beta) \rangle \langle \cos 2(\phi_\beta - \Psi_{RP}) \rangle = k_2 \delta v_2. \tag{3}
\]

Here, \( \delta \equiv \langle \cos(\phi_\alpha - \phi_\beta) \rangle \) represents the charge-dependent two-particle azimuthal correlator and \( k_2 \) is a constant parameter, independent of \( v_2 \), but mainly determined by the kinematics and acceptance of particle detection [24]. As both the charge conservation effect and anisotropic flow are known to be present in heavy ion collisions, the primary goal of this paper is to conduct a systematic investigation of how much of the observed charge-dependent correlations in the data can be accounted for by this mechanism.

Although the background contribution from local charge conservation is well defined in Eq. (3) and has been long recognized [17,20,24], it is still not known to what extent background contributions account for the observed \( \gamma_{112} \) correlator. The main difficulty lies in determining the unknown value of \( k_2 \) in a model-independent way. The other difficulty is to demonstrate directly the linear dependence on \( v_2 \) of \( \gamma_{112} \), which is nontrivial as one has to ensure that the magnetic field, and thus the CME, does not change when selecting events with different \( v_2 \) values. Therefore, selecting events with a quantity that directly relates to the magnitude of \( v_2 \) is essential.

This paper aims to overcome the difficulties mentioned above and achieve a better understanding as to the contribution of the local charge conservation background to the charge-dependent azimuthal correlation data. The results should serve as a new baseline for the search for the CME in heavy ion collisions. Two approaches are employed as outlined below.

1. Higher-order harmonic three-particle correlator: in heavy ion collisions, the charge separation effect from the CME is only expected along the direction of the induced magnetic field normal to the reaction plane, approximated by the second-order event plane \( \Psi_2 \). As the symmetry plane of the third-order Fourier term (“triangular flow” [25]) \( \Psi_3 \) is expected to have a weak correlation with \( \Psi_2 \) [26], the charge separation effect with respect to \( \Psi_3 \) is expected to be negligible. By constructing a charge-dependent correlator with respect to the third-order event plane,

\[
\gamma_{123} \equiv \langle \cos(\phi_\alpha + 2\phi_\beta - 3\Psi_3) \rangle, \tag{4}
\]

charge-dependent background effects unrelated to the CME can be explored. In particular, in the context of the local charge conservation mechanism, the \( \gamma_{123} \) correlator is also expected to have a background contribution, with

\[
\gamma_{123}^{bkg} = k_3 \langle \cos(\phi_\alpha - \phi_\beta) \rangle \langle \cos 3(\phi_\beta - \Psi_3) \rangle = k_3 \delta v_3, \tag{5}
\]

similar to that for the \( \gamma_{112} \) correlator as given in Eq. (3). As the \( k_2 \) and \( k_3 \) parameters mainly depend on particle kinematics and detector acceptance effects, they are expected to be similar, largely independent of harmonic event plane orders. The relation in Eq. (5) can be generalized for all “higher-order harmonic” three-particle correlators, \( \gamma_{1, n-1, 0} = k_n \delta v_n \). Derivation of Eq. (5) as well as generalization to all higher-order harmonics can be found in Appendix A, which follows similar steps as that for Eq. (3) given in Ref. [24]. One caveat here is that when averaging over a wide \( \eta \) and \( p_T \) range, the \( k_n \) value may also depend on the \( \eta \) and \( p_T \) dependence of the \( v_n \) harmonic, which is similar, but not exactly identical, between the \( v_2 \) and \( v_3 \) coefficients [27,28].

By taking the difference of correlators between same- and opposite-sign pairs (denoted as \( \Delta \gamma_{112} \) and \( \Delta \gamma_{123} \) among three particles, and \( \Delta \delta \) between two particles) to eliminate all charge-independent background sources, the following relation is expected to hold if the charge dependence of three-particle correlators is dominated by the effect of local charge...
conservation coupled with the anisotropic flow:

\[
\Delta \gamma_{12} \approx \Delta \gamma_{123} / \Delta v_2.
\]

(6)

Therefore, an examination of Eq. (6) will quantify to what extent the proposed background from charge conservation contributes to the \(\gamma_{12}\) correlator, and will be a critical test of the CME interpretation in heavy ion collisions.

(2) Event shape engineering (ESE): to establish directly a linear relationship between the \(\gamma\) correlators and \(v_n\) coefficients, the ESE technique [29] is employed. In a narrow centrality or multiplicity range (so that the magnetic field does not change significantly), events are further classified based on the magnitude of the event-by-event Fourier harmonic related to the anisotropy measured in the forward rapidity region. Within each event class, the \(\gamma\) correlators and \(v_n\) values are measured and compared to test the linear relationship. A nonzero intercept value of the \(\gamma\) correlators with a linear fit would reflect the strength of the CME.

With a higher luminosity \(p\bar{p}\) run at \(\sqrt{s_{NN}} = 8.16\) TeV and using the high-multiplicity trigger in CMS, the \(p\bar{p}\) data sample gives access to multiplicities comparable to those in peripheral \(p\bar{p}\) collisions, allowing for a detailed comparison and study of the two systems with very different expected CMS contributions in the collisions [21]. Measurements of three-particle correlators \(\gamma_{123}\) and \(\gamma_{12}\), and the two-particle correlator \(\delta\) are presented in different charge combinations as functions of the pseudorapidity (\(\eta\)) difference (\(|\Delta \eta|\)), the transverse momentum (\(p_T\)) difference (\(|\Delta p_T|\)), and the average \(p_T\) of correlated particles (\(\langle p_T \rangle\)). Integrated over \(\eta\) and \(p_T\), the event multiplicity dependence of three- and two-particle correlations is also presented in \(p\bar{p}\) and \(p\bar{p}\) collisions. In \(p\bar{p}\) collisions, the particle correlations are explored separately with respect to the event planes that are obtained using particles with \(4.4 < |\eta| < 5.0\) from the \(p\)- and \(\bar{p}\)-going beam directions. The ESE analysis is performed for \(\gamma_{112}\) as a function of \(v_2\) in both \(p\bar{p}\) and \(p\bar{p}\) collisions.

This paper is organized as follows. After a brief description of the detector and data samples in Sec. II, the event and track selections are discussed in Sec. III, followed by the discussion of the analysis technique in Sec. IV. The results are presented in Sec. V, and the paper is summarized in Sec. VI.

II. DETECTOR AND DATA SAMPLES

The central feature of the CMS apparatus is a superconducting solenoid of 6 m internal diameter, providing a magnetic field of 3.8 T. Within the solenoid volume, there are four primary subdetectors, including a silicon pixel and strip tracker detector, a lead tungstate crystal electromagnetic calorimeter (ECAL), and a brass and scintillator hadron calorimeter (HCAL), each composed of a barrel and two endcap sections. The silicon tracker measures charged particles within the range \(|\eta| < 2.5\). Iron and quartz-fiber Cherenkov hadron forward (HF) calorimeters cover the range \(2.9 < |\eta| < 5.2\). The HF calorimeters are constituted of towers, each of which is a two-dimensional cell with a granularity of 0.5 units in \(\eta\) and 0.349 radians in \(\phi\). For charged particles with \(1 < p_T < 10\) GeV and \(|\eta| < 1.4\), the track resolutions are typically 1.5% in \(p_T\) and 25–90 (45–150) \(\mu\)m in the transverse (longitudinal) impact parameter [30]. A detailed description of the CMS detector, together with a definition of the coordinate system used and the relevant kinematic variables, can be found in Ref. [31].

The \(p\bar{p}\) data at \(\sqrt{s_{NN}} = 8.16\) TeV used in this analysis were collected in 2016, and correspond to an integrated luminosity of 186 nb\(^{-1}\). The beam energies are 6.5 TeV for the protons and 2.56 TeV per nucleon for the lead nuclei. The data were collected in two different run periods: one with the protons circulating in the clockwise direction in the LHC ring, and one with them circulating in the counterclockwise direction. By convention, the proton beam rapidity is taken to be positive when combining the data from the two run periods. A subset of PbPb data at \(\sqrt{s_{NN}} = 5.02\) TeV collected in 2015 (30–80% centrality, where centrality is defined as the fraction of the total inelastic cross section, with 0% denoting the most central collisions) is used. The PbPb data were reprocessed using the same reconstruction algorithm as the \(p\bar{p}\) data, in order to compare directly the two colliding systems at similar final-state multiplicities. The three-particle correlator, \(\gamma_{112}\), data for \(p\bar{p}\) collisions at \(\sqrt{s_{NN}} = 8.16\) TeV are compared to those previously published at \(\sqrt{s_{NN}} = 5.02\) TeV [21] to examine any possible collision energy dependence. Because of statistical limitations, new analyses of higher-order harmonic three-particle correlator and event shape engineering introduced in this paper cannot be performed with the 5.02-TeV \(p\bar{p}\) Pb data.

III. SELECTION OF EVENTS AND TRACKS

The event reconstruction, event selections, and the triggers, including the dedicated triggers to collect a large sample of high-multiplicity \(p\bar{p}\) events at \(\sqrt{s_{NN}} = 8.16\) TeV, are similar to those used in previous CMS particle correlation measurements at lower energies [28,32–34], as discussed below. For PbPb events, they are identical to those in Ref. [21].

Minimum bias \(p\bar{p}\) events at 8.16 TeV were selected by requiring energy deposits in at least one of the two HF calorimeters above a threshold of approximately 1 GeV and the presence of at least one track with \(p_T > 0.4\) GeV in the pixel tracker. In order to collect a large sample of high-multiplicity \(p\bar{p}\) collisions, a dedicated trigger was implemented using the CMS level-1 (L1) and high-level trigger (HLT) systems. At L1, the total number of towers of ECAL+HCAL above a threshold of \(0.5\) GeV in transverse energy \((E_T)\) was required to be greater than a given threshold (120 and 150 towers), where a tower is defined by \(\Delta \eta \times \Delta \phi = 0.087 \times 0.087\) radians. Online track reconstruction for the HLT was based on the same offline iterative tracking algorithm to maximize the trigger efficiency. For each event, the vertex reconstructed with the greatest number of tracks was selected. The number of tracks with \(|\eta| < 2.4, p_T > 0.4\) GeV, and a distance of closest approach less than 0.12 cm to this vertex, was determined for each event and required to exceed a certain threshold (120, 150, 185, 250).

In the offline analysis of \(p\bar{p}\) (PbPb) collisions, hadronic events are selected by requiring the presence of at least one (three) energy deposit(s) greater than 3 GeV in each of the two HF calorimeters. Events are also required to contain a primary vertex within 15 cm of the nominal interaction point along the
beam axis and 0.15 cm in the transverse direction. In the pPb data sample, the average pileup (number of interactions per bunch crossing) varied between 0.1 to 0.25 pPb interactions per bunch crossing. A procedure similar to that described in Ref. [28] is used for identifying and rejecting pileup events. It is based on the number of tracks associated with each reconstructed vertex and the distance between multiple vertices. The pileup in PbPb data is negligible.

For track selections, the impact parameter significance of the track with respect to the primary vertex in the direction along the beam axis and in the transverse plane, \( d_{\text{imp}} / \sigma (d_{\text{imp}}) \), is required to be less than 3. The relative uncertainty in \( p_T \), \( \sigma (p_T) / p_T \), must be less than 10%. Primary tracks, i.e., tracks that originate at the primary vertex and satisfy the high-purity criteria of Ref. [30], are used to define the event charged-particle multiplicity \( N_{\text{ch}}^{\text{offline}} \). To perform correlation measurements, each track is also required to leave at least one hit in one of the three layers of the pixel tracker. Only tracks with \( |\eta| < 2.4 \) and \( p_T > 0.3 \text{ GeV} \) are used in this analysis to ensure high tracking efficiency.

The pPb and PbPb data are compared in classes of \( N_{\text{ch}}^{\text{offline}} \), where primary tracks with \( |\eta| < 2.4 \) and \( p_T > 0.4 \text{ GeV} \) are counted. To compare with results from other experiments, the PbPb data are also analyzed based on centrality classes for the 30–80% centrality range.

IV. ANALYSIS TECHNIQUE

The analysis technique of three-particle correlations employed in this paper is based on that established in Ref. [21], with the extension of charge-dependent two-particle correlations, higher-order harmonic three-particle correlations, and correlation studies in different event shape classes (i.e., ESE analysis). The details are outlined below.

A. Calculations of two- and three-particle correlators

Without directly reconstructing the event plane, the expression given in Eq. (2) can be alternatively evaluated using a three-particle correlator with respect to a third particle \([11,12]\), \( \langle \cos (\phi_\alpha + \phi_\beta - 2\phi_i) \rangle / v_{2,c} \), where \( v_{2,c} \) is the elliptic flow anisotropy of particle \( c \) with inclusive charge sign. The three-particle correlator is measured via the scalar-product method of \( Q \) vectors. A complex \( Q \) vector for each event is defined as \( Q_n \equiv \sum_{i=1}^M w_i e^{i\phi_i} / W \), where \( \phi_i \) is the azimuthal angle of particle \( i \), \( n \) is the Fourier harmonic order, \( M \) is the number of particles in the \( Q \) calculation in each event, and \( w_i \) is a weight assigned to each particle for efficiency correction, which is derived from a simulation using the HIJING event generator [35]. The \( W = \sum_{i=1}^M w_i \) represents the weight of the \( Q \) vector. In this way, the three-particle correlator can be expressed in terms of the product of \( Q \) vectors, i.e., \( Q_{1,\alpha} \) and \( Q_{1,\beta} \), when particles \( \alpha \) and \( \beta \) are chosen from different detector phase-space regions or carry different charge signs,

\[
\gamma_{112} = \frac{\langle \cos (\phi_\alpha + \phi_\beta - 2\phi_i) \rangle}{v_{2,c}} = \frac{\langle Q_{1,\alpha} Q_{1,\beta} Q_{2,\text{HF}^\pm}^* \rangle}{\sqrt{\langle Q_{2,\text{HF}^+} Q_{2,\text{HF}^-} \rangle \langle Q_{2,\text{HF}^+} Q_{2,\text{HF}^-} \rangle}}.
\]

where the angle brackets on the right-hand side denote an event average of the \( Q \)-vector products, weighted by the product of their respective total weights \( W \). Here \( Q_{2,\text{HF}^\pm} \) is the charge inclusive \( Q_2 \) vector of all particles in the tracker region, and \( Q_{2,\text{HF}^\pm} \) denotes the \( Q_2 \)-vector for particles \( c \) detected in the HF towers. When particles \( \alpha \) and \( \beta \) are of the same sign and share the same phase space region (denoted as \( \alpha = \beta \)), an extra term is needed to remove the contribution of a particle pairing with itself, so evaluation of the three-particle correlator is modified as

\[
\gamma_{112} = \frac{\langle \cos (\phi_\alpha + \phi_\beta - 2\phi_i) \rangle}{v_{2,c}} = \frac{\langle Q_{1,\alpha} Q_{1,\beta} Q_{2,\text{HF}^\pm}^* \rangle}{\sqrt{\langle Q_{2,\text{HF}^+} Q_{2,\text{HF}^-} \rangle \langle Q_{2,\text{HF}^+} Q_{2,\text{HF}^-} \rangle}}.
\]

where the \( \gamma_{112} \) is defined as

\[
Q_{112} = \frac{\langle \sum_{i=1}^M w_i \cos (\phi_i) \rangle^2 - \sum_{i=1}^M w_i^2 e^{2i\phi_i}}{\langle \sum_{i=1}^M w_i \rangle^2 - \sum_{i=1}^M w_i^2},
\]

and the denominator of Eq. (9) is the respective event weight associated with \( Q_{112} \).

In the numerators of Eqs. (7) and (8), the particles \( \alpha \) and \( \beta \) are identified in the tracker, with \( |\eta| < 2.4 \) and \( 0.3 < p_T < 3 \text{ GeV} \), and are assigned a weight factor \( w_i \) to correct for tracking inefficiency. The particle \( c \) is selected by using the tower energies and positions in the HF calorimeters with \( 4.4 < |\eta| < 5.0 \). This choice of \( \eta \) range for the HF towers imposes an \( \eta \) gap of at least two units with respect to particles \( \alpha \) and \( \beta \) from the tracker, to minimize possible short-range correlations. To account for any occupancy effect of the HF detectors resulting from the large granularities in \( \eta \) and \( \phi \), each tower is assigned a weight factor \( w_i \) corresponding to its \( E_T \) value when calculating the \( Q \) vector. The denominator of the right-hand side of Eqs. (7) and (8) corresponds to the \( v_{2,c} \) using the scalar-product method [11,12], with \( Q_{2,\text{HF}^\pm} \) denoting \( Q_2 \) vectors obtained from the tracker and the two HF detectors (positive and negative \( \eta \) side) with the same kinematic requirements as for the numerator. The three-particle correlator is evaluated for particles \( \alpha \) and \( \beta \) carrying the same sign (SS) and opposite sign (OS). The SS combinations, \( (+,+) \) and \( (-,-) \), give consistent results and are therefore combined. For pPb collisions, the three-particle correlator is also measured with particle \( c \) from \( \text{HF}^+ \) and \( \text{HF}^- \), corresponding to the \( p \)- and Pb-going direction, respectively. For symmetric pPb collisions, the results from \( \text{HF}^+ \) and \( \text{HF}^- \) are consistent with each other and thus combined.

The higher-order harmonic three-particle correlator, \( \gamma_{123} \), defined in Eq. (4), is evaluated in exactly the same way as the \( \gamma_{112} \) correlator as follows when particles \( \alpha \) and \( \beta \) do not overlap,

\[
\gamma_{123} = \frac{\langle \cos (\phi_\alpha + 2\phi_\beta - 3\phi_i) \rangle}{v_{3,c}} = \frac{\langle Q_{1,\alpha} Q_{2,\beta} Q_{3,\text{HF}^\pm}^* \rangle}{\sqrt{\langle Q_{3,\text{HF}^+} Q_{3,\text{HF}^-} \rangle \langle Q_{3,\text{HF}^+} Q_{3,\text{HF}^-} \rangle}}.
\]
the $\gamma_{123}$ can be evaluated via
\[
\gamma_{123} = \frac{\langle \cos(\phi_\alpha + 2\phi_\beta - 3\phi_c) \rangle}{v_{3,c}} = \frac{\langle Q_{123} Q_{1,\text{HF},c}^* \rangle}{\sqrt{(Q_{1,\text{HF},c}^*, Q_{2,\text{HF},c}^*) (Q_{1,\text{HF},c}, Q_{2,\text{HF},c})}},
\]
where $Q_{123}$ is defined as
\[
Q_{123} \equiv \left( \sum_{i=1} w_i e^{i\phi} \sum_{j=1} w_j e^{i2\phi} - \sum_{i=1} w_i^2 \right) / \left( \sum_{i=1} w_i^2 - \sum_{i=1} w_i^2 \right),
\]
and the respective event weight associated with $Q_{123}$ is the denominator of Eq. (12).

Similarly, the charge-dependent two-particle correlator, $\delta \equiv \langle \cos(\phi_\alpha - \phi_\beta) \rangle$, is also evaluated with $Q$ vectors as $\delta = \langle Q_{1,a} Q_{1,b}^* \rangle$ when particles $\alpha$ and $\beta$ are chosen from different detector phase-space regions or have opposite signs, or otherwise,
\[
\delta = \left( \sum_{i=1} w_i e^{i\phi} \sum_{j=1} w_j e^{-i\phi} - \sum_{i=1} w_i^2 \right) / \left( \sum_{i=1} w_i^2 - \sum_{i=1} w_i^2 \right),
\]
and the respective event weight is the denominator of Eq. (13).

The effect of the nonuniform detector acceptance is corrected by evaluating the cumulants of $Q$-vector products [36]. While the correction is found to be negligible for the $\gamma_{112}$ and $\delta$ correlators, there is a sizable effect of 5–10% correction to the $\gamma_{123}$ correlator.

**B. Event shape engineering**

In the ESE analysis, within each multiplicity range of pPb or centrality range of PbPb data, events are divided into different $q_2$ classes, where $q_2$ is defined as the magnitude of the $Q_2$ vector. In this analysis, the $q_2$ value is calculated from one side of the HF region within the range $3 < \eta < 5$ for both pPb and PbPb collisions (weighted by the tower $E_T$), where in pPb collisions only the p-going side of HF is used because of the poor resolution from a relatively low charged-particle multiplicity on the proton-going side. In each $q_2$ class, the $v_2$ harmonic is measured with the scalar product method using a common resolution term ($v_{2,c}$) as in the $\gamma_{112}$ correlator. Therefore, the $v_2$ from the tracker region can be expressed in terms of the $Q$-vectors as
\[
v_2 = \frac{\langle Q_{1,\text{HF},c} Q_{2,\text{HF},c}^* \rangle}{\sqrt{(Q_{1,\text{HF},c}, Q_{2,\text{HF},c}) (Q_{1,\text{HF},c}, Q_{2,\text{HF},c})}},
\]
where particles from the HF are selected from the same region as particle $c$ in the $\gamma_{112}$ correlator.

In PbPb collisions, the particle $c$ in the $\gamma_{112}$ correlator is taken from the HF detector that is at the opposite $\eta$ side to the one used to calculate $q_2$. However, the results are in good agreement with those where the particle $c$ for $\gamma_{112}$ and $q_2$ is measured from the same side of the HF detector, which can be found in Appendix B. In pPb collisions, the particle $c$ in the $\gamma_{112}$ correlator with respect to the Pb- and p-going sides is studied, when $q_2$ is measured only in the Pb-going side. The results are found to be independent of the side in which the particle $c$ is detected.

In Fig. 1, the HF $q_2$ distributions are shown for pPb and PbPb collisions in the multiplicity range $185 \leq N_{\text{off}} \leq 250$, where most of the high-multiplicity pPb events were recorded by the high-multiplicity trigger in this range. As indicated by the vertical dashed lines, the distribution is divided into several intervals with each corresponding to a fraction of the full distribution, where 0–1% represents the highest $q_2$ class. For each $q_2$ class, the three-particle $\gamma_{112}$ is calculated with the default kinematic regions for particles $\alpha$, $\beta$, and $c$.
and the $v_2$ harmonics from the tracker ($|\eta| < 2.4$) are also obtained by the scalar-product method [37]. The pPb and PbPb results are presented in Sec. V for both SS and OS pairs, as well as the differences found for the two charge combinations.

In Fig. 2, the $v_2$ values for tracker particles as a function of the average $q_2$ in each HF class are shown. A proportionality close to linear is seen, indicating the two quantities are strongly correlated because of the initial-state geometry [38].

C. Systematic uncertainties

The absolute systematic uncertainties of the two-particle correlator $\delta$, and three-particle correlators $\gamma_{112}$ and $\gamma_{123}$, have been studied. Varying the $d_1/\sigma(d_1)$ and $dT/\sigma(dT)$ from less than 3 (default) to less than 2 and 5, and the $\sigma(pT)/pT < 10\%$ (default) to $\sigma(pT)/pT < 5\%$, together yield the systematic uncertainties of $\pm 1.0 \times 10^{-5}$ for the $\gamma_{112}$, $\pm 4.0 \times 10^{-5}$ for the $\gamma_{123}$, and $\pm 1.0 \times 10^{-4}$ for the $\delta$ correlator. The longitudinal primary vertex position ($V_z$) has been varied, using ranges $|V_z| < 3$ cm and $3 < |V_z| < 15$ cm, where the differences with respect to the default range $|V_z| < 15$ cm are $\pm 1.0 \times 10^{-5}$ for the $\gamma_{112}$, $\pm 3.0 \times 10^{-5}$ for the $\gamma_{123}$, and $\pm 1.0 \times 10^{-4}$ for the $\delta$ correlator, taken as the systematic uncertainty. In the pPb collisions only, using the lower threshold of the high-multiplicity trigger with respect to the default trigger, a systematic uncertainty of $\pm 3.0 \times 10^{-5}$ are yielded for all three correlators, which accounts for the possible trigger bias from the inefficiency of the default trigger around the threshold. In the pPb data sample, the average pileup can be as high as 0.25 and therefore the systematic effects from pileup have been evaluated. The full sample has been split into four different sets of events with different average pileup, according to their instantaneous luminosity during each run. The systematic effects for $\gamma_{112}$ and $\delta$ have been found to be $\pm 1.0 \times 10^{-5}$, and for $\gamma_{123}$ is found to be $\pm 3.0 \times 10^{-5}$.

A final test of the analysis procedures is done by comparing “known” charge-dependent signals based on the EPOS event generator [39] to those found after events are passed through a GEANT4 [40,41] simulation of the CMS detector response. Based on this test, a systematic uncertainty of $\pm 2.5 \times 10^{-5}$ is assigned for the $\gamma_{112}$, $\pm 4.0 \times 10^{-5}$ for the $\gamma_{123}$, and $\pm 5.0 \times 10^{-4}$ for the $\delta$ correlators, by taking the difference in the correlators between the reconstructed and the generated level. Note that this uncertainty for the $\delta$ correlator is based on differential variables, where the uncertainty covers the maximum deviation from the closure test. For results that averaged over $|\Delta \eta| < 1.6$, the systematic uncertainty is found to be $\pm 2.0 \times 10^{-4}$ when directly evaluating the average. The tracking efficiency and acceptance of positively and negatively charged particles have been evaluated separately, and the difference has been found to be negligible. All sources of systematic uncertainty are uncorrelated and added in quadrature to obtain the total absolute systematic uncertainty. No dependence of the systematic uncertainties on the sign combination, multiplicity, $\Delta \eta$, $\Delta p_T$, or average-$p_T$ is found. The systematic uncertainties in our results are point-to-point correlated. In pPb collisions, the systematic uncertainty is also observed to be independent of particle $c$ pointing to the Pb- or $p$-going direction, and thus it is quoted to be the same for these two situations. The systematic uncertainties are summarized in Table I.

V. RESULTS

A. Charge-dependent two- and three-particle correlators

Measurements of the charge-dependent three-particle ($\gamma_{112}$, $\gamma_{123}$) and two-particle ($\delta$) correlators are shown in Fig. 3 as functions of the pseudorapidity difference ($|\Delta \eta| \equiv |\eta_\alpha - \eta_\beta|$) between SS and OS particles $\alpha$ and $\beta$, in the multiplicity range $185 \leq N_{\text{offline}} < 250$ for pPb collisions at $\sqrt{s_{NN}} = 8.16$ TeV and PbPb collisions at 5.02 TeV. The SS and OS of $\delta$ correlators are shown with different markers to differentiate the two-charge combinations. The $p$-going direction. The $p$-Pb data are obtained with particle $c$ in the Pb- and $p$-going sides separately. The
obtained with particle \( \gamma \)\( p\) and PbPb collisions at 5.02 TeV (right). The flow [the first-order azimuthal anisotropy in Eq. (1)] and the multiset correlations that are charge independent, such as directed flow, correspond to the centrality range 60–65%. Similar to the observation reported in Ref. [21], the three-particle correlators, \( \gamma_{123} \) (middle), and two-particle correlator, \( \delta \) (lower), as a function of \( |\Delta \eta| \) for \( 185 < N_{\text{trk}}^{\text{offline}} < 250 \) in pPb collisions at \( \sqrt{s_{NN}} = 8.16 \) TeV (left) and PbPb collisions at 5.02 TeV (right). The pPb results obtained with particle c in Pb-going (solid markers) and p-going (open markers) sides are shown separately. The SS and OS two-particle correlators are denoted by different markers for both pPb and PbPb collisions. Statistical and systematic uncertainties are indicated by the error bars and shaded regions, respectively.

FIG. 3. The SS and OS three-particle correlators, \( \gamma_{123} \) (middle), and two-particle correlator, \( \delta \) (lower), as a function of \( |\Delta \eta| \) for \( 185 < N_{\text{trk}}^{\text{offline}} < 250 \) in pPb collisions at \( \sqrt{s_{NN}} = 8.16 \) TeV (left) and PbPb collisions at 5.02 TeV (right). The pPb results obtained with particle c in Pb-going (solid markers) and p-going (open markers) sides are shown separately. The SS and OS two-particle correlators are denoted by different markers for both pPb and PbPb collisions. Statistical and systematic uncertainties are indicated by the error bars and shaded regions, respectively.

Similar to the observation reported in Ref. [21], the three-particle correlator \( \gamma_{123} \) [Figs. 3(a) and 3(b)] and \( \gamma_{123} \) [Figs. 3(c) and 3(d)] correlators show a charge dependence for \( |\Delta \eta| < 250 \) in pPb collisions at \( \sqrt{s_{NN}} = 8.16 \) TeV (left) and PbPb collisions at 5.02 TeV (right) systems. Little collision energy dependence of the \( \gamma_{123} \) data for pPb collisions is found from \( \sqrt{s_{NN}} = 5.02 \) TeV to 8.16 TeV within uncertainties (as will be shown later in Figs. 6 and 8 as a function of event multiplicity). For \( |\Delta \eta| > 1.6 \), the SS and OS correlators converge to a common value, which is weakly dependent on \( |\Delta \eta| \) out to about 4.8 units. In pPb collisions, the \( \gamma_{112} \) correlator obtained with particle c from the p-going side is shifted toward more positive values than that from the Pb-going side by approximately the same amount for both the SS and OS pairs. This trend is reversed for the higher-order-harmonic \( \gamma_{123} \) correlator, where the Pb-going side data are more positive than the p-going side data. The Pb-going side results for the \( \gamma_{112} \) correlator for the pPb collisions are of similar magnitude as the results for PbPb collisions, although a more pronounced peak structure at small \( |\Delta \eta| \) is observed in pPb collisions. The common shift of SS and OS correlators between the p- and Pb-going side reference (c) particle may be related to sources of correlation that are charge independent, such as directed flow [the first-order azimuthal anisotropy in Eq. (1)] and the momentum conservation effect, the latter being sensitive to the difference in multiplicity between p- and Pb-going directions.

The two-particle \( \delta \) correlators [Figs. 3(e) and 3(f)] for both SS and OS pairs also show a decreasing trend as \( |\Delta \eta| \) increases and converge to the same values at \( |\Delta \eta| \approx 1.6 \), similar to that for the three-particle correlators. The values of both OS and SS \( \delta \) correlators are found to be larger in pPb than in PbPb collisions at similar multiplicities. As the \( \delta \) correlator is sensitive to short-range jetlike correlations, reflected by the low-\( |\Delta \eta| \) region, this effect may be related to the higher-\( p_{T} \) jets or clusters in pPb compared to PbPb collisions at similar multiplicities, as suggested in Ref. [28], because of short-range two-particle \( \Delta \eta \)–\( \Delta \phi \) correlations.

To provide more detailed information on the particle \( p_{T} \) dependence of the correlations, the \( \gamma_{112}, \gamma_{123}, \) and \( \delta \) correlators are measured as functions of the \( p_{T} \) difference (\( |\Delta p_{T}| \equiv |p_{T,a} - p_{T,b}| \)) and average (\( \langle p_{T} \rangle \equiv (p_{T,a} + p_{T,b})/2 \)) of the SS and OS pairs in pPb and PbPb collisions, and shown in Figs. 4 and 5. The \( |\Delta p_{T}| \)– and \( \langle p_{T} \rangle \)-dependent results are averaged over the full \( |\eta| < 2.4 \) range. In particular, the charge-dependent correlations from the CME are expected to be strongest in the low-\( p_{T} \) region [6].

For all correlators, similar behaviors between pPb and PbPb data are again observed. The trends in \( |\Delta p_{T}| \) for \( \gamma_{112} \) and \( \gamma_{123} \) correlators seem to be opposite. The \( \gamma_{112} \) correlator increases...
as a function of $|\Delta p_T|$, while a decreasing trend is seen for
the $\gamma_{123}$ correlator up to $|\Delta p_T| \approx 2$ GeV, where $\gamma_{123}$ becomes
constant in $|\Delta p_T|$. The opposite behavior observed between
the $\gamma_{112}$ and $\gamma_{123}$ correlators is related to back-to-back jetlike
correlations, which give a positive (negative) contribution to
even- (odd-)order Fourier harmonics [42]. The $\delta$ correlators
decrease monotonically as functions of $|\Delta p_T|$ for both SS and
OS pairs in $p$Pb and PbPb collisions. This trend of decreasing
for $\delta$ is consistent with the expectation from either transverse
momentum conservation or back-to-back jet correlations [19].

In terms of the $\overline{p}_T$- dependence in Fig. 5, all three correlators
for both SS and OS pairs show very similar behaviors in the
low-$\overline{p}_T$ region, which is likely a consequence of the same
physical origin. However, an opposite trend starts emerging
at $\overline{p}_T \approx 1.6$ GeV, most evidently for $\gamma_{112}$ and $\delta$. Within the
0.3 < $p_T$ < 3 GeV range, as $\overline{p}_T$ increases toward 3 GeV, both
particles of a pair tend to be selected with a high-$p_T$ value,
while for low-$\overline{p}_T$ or any $|\Delta p_T|$ values, the pair usually consists
of at least one low-$p_T$ particle. This may be the reason for a
different trend seen at high $\overline{p}_T$. The qualitative behavior of the
data is captured by a multiphase transport model [43,44]. In
Appendix C, all three correlators as functions of $|\Delta \eta|$, $\Delta p_T$, and
$\overline{p}_T$ in different multiplicity and centrality ranges in $p$Pb
and PbPb collisions can be found.

To explore the multiplicity or centrality dependence of the
three- and two-particle correlators, an average of the data is
taken over $|\Delta \eta| < 1.6$, corresponding to the region in Fig. 3
which exhibits charge dependence. The average over $|\Delta \eta| < 1.6$
is weighted by the density of particle pairs in $|\Delta \eta|$, and all
further plots averaged over $|\Delta \eta| < 1.6$ are weighted similarly.
The resulting $|\Delta \eta|$-averaged data of $\gamma_{112}$, $\gamma_{123}$, and $\delta$ are shown
in Fig. 6 for both OS and SS pairs, as functions of $N_{\text{trk}}^{\text{offline}}$
for PbPb collisions at $\sqrt{s_{NN}} = 8.16$ TeV (particle $c$ from the
Pb-going side) and PbPb collisions at 5.02 TeV. Previously
published PbPb data at 5.02 TeV are also shown for comparison
[21]. The centrality scale on the top of Fig. 6 relates to the PbPb
experimental results. Up to $N_{\text{trk}}^{\text{offline}} = 400$, the $p$Pb and PbPb

FIG. 5. The SS and OS three-particle correlators, $\gamma_{112}$ (upper) and
$\gamma_{123}$ (middle), and two-particle correlator, $\delta$ (lower), as a function of
$\overline{p}_T$ for $185 < N_{\text{trk}}^{\text{offline}} < 250$ in $p$Pb collisions at $\sqrt{s_{NN}} = 8.16$ TeV
(left) and PbPb collisions at 5.02 TeV (right). The $p$Pb results
obtained with particle $c$ in Pb-going (solid markers) and $p$-going (open
markers) sides are shown separately. The SS and OS two-particle
correlators are denoted by different markers for both $p$Pb and PbPb
collisions. Statistical and systematic uncertainties are indicated by the
error bars and shaded regions, respectively.

FIG. 6. The SS and OS three-particle correlators, $\gamma_{112}$ (upper) and
$\gamma_{123}$ (middle), and two-particle correlator, $\delta$ (lower), averaged
over $|\Delta \eta| < 1.6$ as a function of $N_{\text{trk}}^{\text{offline}}$ in $p$Pb collisions at $\sqrt{s_{NN}} = 8.16$ TeV and PbPb collisions at 5.02 TeV. The SS and OS two-particle
correlators are denoted by different markers for PbPb collisions.
The results of $\gamma_{112}$ for PbPb collisions at 5.02 TeV from the CMS
Collaboration [21] are also shown for comparison. Statistical and
systematic uncertainties are indicated by the error bars and shaded
regions, respectively.
5.02 TeV. The functions of $\Delta_1p$ and $\Delta_1s$ collisions at OS three-particle correlators as functions of centrality in PbPb of the signal is not related to the CME. The results of SS and PbPb collisions again indicates that the dominant contribution correlator magnitude and multiplicity dependence of the three-particle the inverse of event multiplicity [12]. The notably similar dependence is understood as a dilution effect that falls with increases. Part of the observed multiplicity (or centrality) results are measured in the same $N_{\text{offline}}$ ranges. The new pPb data at 8.16 TeV extend the multiplicity reach further than the previously published pPb data at 5.02 TeV (which stopped at $N_{\text{offline}} \approx 300$).

Within the uncertainties, the SS and OS $\gamma_{112}$ correlators in pPb and PbPb collisions exhibit the same magnitude and trend as functions of event multiplicity. The pPb data are independent of collision energy from 5.02 to 8.16 TeV at similar multiplicities. This justifies the comparison of new pPb data and PbPb data at somewhat different energies. For both pPb and PbPb collisions, the OS correlator reaches a value close to zero for $N_{\text{offline}} > 200$, while the SS correlator remains negative, but the magnitude gradually decreases as $N_{\text{offline}}$ increases. Part of the observed multiplicity (or centrality) dependence is understood as a dilution effect that falls with the inverse of event multiplicity [12]. The notably similar magnitude and multiplicity dependence of the three-particle correlator $\gamma_{112}$ observed in pPb collisions relative to that in PbPb collisions again indicates that the dominant contribution of the signal is not related to the CME. The results of SS and OS three-particle correlators as functions of centrality in PbPb collisions at $\sqrt{s_{\text{NC}}} = 5.02$ TeV are also found to be consistent with the results from lower energy $AA$ collisions [12,16]. However, values of $\gamma_{123}$ correlators between pPb and PbPb are observed to be different, unlike those for $\gamma_{112}$ correlators. As the CME contribution to $\gamma_{123}$ is not expected, the data suggest different properties of backgrounds in pPb and PbPb systems. If the $\gamma_{112}$ correlator in pPb data is expected to be background dominated, as argued earlier, the similarity found to the PbPb data in $\gamma_{112}$ requires further understanding. The two-particle $\delta$ correlators show a similar trend in multiplicity between pPb and PbPb systems, but a larger splitting between OS and SS pairs is observed in pPb than in PbPb data.

To eliminate sources of correlations that are charge independent (e.g., directed flow, $v_1$) and to explore a possible charge separation effect generated by the CME or charge-dependent background correlations, the differences of three-particle correlators $\Delta\gamma_{112}$ and $\Delta\gamma_{123}$ and two-particle correlator $\Delta\delta$ between OS and SS are shown in Fig. 7 as functions of $|\Delta\eta|$, $|\Delta p_T|$, and $p_T$ in the multiplicity range 185 $\leq N_{\text{offline}} < 250$ for pPb collisions at $\sqrt{s_{\text{NC}}} = 8.16$ TeV and PbPb collisions at 5.02 TeV.

After taking the difference, the three-particle correlators $\Delta\gamma_{112}$ and $\Delta\gamma_{123}$ in pPb collisions with particle $c$ from either the $p$- or Pb-going side and in PbPb collisions show nearly identical values, except in the high $p_T$ region. Note that for OS and SS correlators separately, this similarity between pPb and PbPb is only observed for the $\gamma_{112}$ correlator. As a function of
The results of regions, respectively. Statistical and systematic uncertainties are indicated by the error bars and shaded correlations between OS and SS are found for the $p$ Pb system. The similarity seen between high-multiplicity $p$ Pb and peripheral PbPb collisions for both $\Delta \gamma_{12}$ and $\Delta \gamma_{123}$ further challenges the attribution of the observed charge-dependent correlations to the CME. The two-particle correlator $\Delta \delta$, on the other hand, is found to show a larger value in $p$ Pb than in PbPb collisions.

The differences of three-particle correlators $\Delta \gamma_{12}$ and $\Delta \gamma_{123}$ and two-particle correlator $\Delta \delta$ between OS and SS are shown in Fig. 8 as functions of $N_{\text{trk}}$ averaged over $|\Delta \eta| < 1.6$ for $p$ Pb collisions at $\sqrt{s_{NN}} = 8.16$ TeV and PbPb collisions at 5.02 TeV. For comparison, previously published $p$ Pb data at 5.02 TeV are also shown [21]. Similar to those shown in Fig. 7, the observed difference between OS and SS pairs in $\Delta \gamma_{12}$ and $\Delta \gamma_{123}$ is strikingly similar in $p$ Pb and PbPb collisions over the entire overlapping multiplicity range (and also independent of collision energy for $\Delta \gamma_{12}$ in $p$ Pb), while higher values of an OS-SS difference in $\Delta \delta$ are found for the $p$ Pb system.

To check if the mechanism of local charge conservation coupled with anisotropic flow can explain the observed charge dependence of the $\Delta \gamma_{12}$ and $\Delta \gamma_{123}$ correlators, the relation in Eq. (6) is used. The ratios of $\Delta \gamma_{112}$ and $\Delta \gamma_{123}$ to the product of $v_2$ and $\delta$, as functions of event multiplicity in $p$ Pb and PbPb collisions. The $v_2$ and $v_3$ values for particles $\alpha$ or $\beta$ are calculated with the scalar-product method with respect to the particle $c$. In $p$ Pb collisions, only results with the $p$-going direction are shown because the $p$-going direction data lack statistical

FIG. 8. The difference of the OS and SS three-particle correlators $\gamma_{12}$ (upper) and $\gamma_{123}$ (middle) and two-particle correlator $\delta$ (lower) averaged over $|\Delta \eta| < 1.6$ as a function of $N_{\text{trk}}$ in $p$ Pb collisions at $\sqrt{s_{NN}} = 8.16$ TeV and PbPb collisions at 5.02 TeV. The $p$ Pb results are obtained with particle $c$ from $p$- and $p$-going sides separately. The $\Delta \delta$ correlator is denoted by a different marker for $p$ Pb collisions. The results of $\gamma_{112}$ for $p$ Pb collisions at 5.02 TeV from the CMS Collaboration [21] are also shown for comparison. Statistical and systematic uncertainties are indicated by the error bars and shaded regions, respectively.

FIG. 9. The ratio of $\Delta \gamma_{12}$ and $\Delta \gamma_{123}$ to the product of $v_2$ and $\delta$, averaged over $|\Delta \eta| < 1.6$, in $p$ Pb collisions for the $p$-going direction at $\sqrt{s_{NN}} = 8.16$ TeV (upper) and PbPb collisions at 5.02 TeV (lower). Statistical and systematic uncertainties are indicated by the error bars and shaded regions, respectively.
As a result, the product of $\Delta \gamma_{112}$ and $\Delta \gamma_{123}$ to the product of $v_n$ and $\delta$, as functions of $|\Delta \eta|$ (left), $\Delta p_T$ (middle), and $|\vec{p}_T|$ (right) for $185 \leq N_{trk}^{\text{offline}} < 250$ in pPb collisions at $\sqrt{s_{NN}} = 8.16$ TeV (upper) and PbPb collisions at 5.02 TeV (lower). Statistical and systematic uncertainties are indicated by the error bars and shaded regions, respectively.

The ratios shown in Fig. 9 for both systems are found to be similar between $n = 2$ and $n = 3$, on average with values slightly less than 2. This observation indicates that the measured charge dependence of three-particle correlators is consistent with mostly being dominated by charge-dependent two-particle correlations (e.g., from local charge conservation) coupled with the anisotropic flow $v_n$. For a given $n$ value, the ratios are also similar between pPb and PbPb collisions (and may reflect similar particle kinematics and acceptances), and approximately constant as functions of event multiplicity. Notably, the $\Delta \delta$ in Fig. 8 are different between the pPb and PbPb systems. However, the anisotropic flow harmonics $v_n$ are larger for PbPb collisions than for pPb collisions [28]. As a result, the product of $\Delta \delta$ and $v_n$ leads to similar values of $\Delta \gamma_{112}$ and $\Delta \gamma_{123}$ correlators between the pPb and PbPb systems, implying the $k_2$ is similar to $k_3$.

The ratios of $\Delta \gamma_{112}$ and $\Delta \gamma_{123}$ to the product of $\Delta \delta$ and $v_n$ can also be studied as functions of $|\Delta \eta|$, $\Delta p_T$, and $|\vec{p}_T|$ in pPb and PbPb collisions, as shown in Fig. 10 for the multiplicity range of $185 \leq N_{trk}^{\text{offline}} < 250$. Here, the $v_n$ are calculated as the average $v_n$ of particles $\alpha$ and $\beta$, $v_n = (v_{n,\alpha} + v_{n,\beta})/2$ [based on the relation derived in Eq. (A5) in Appendix A], and are weighted by the number of pairs of particles $\alpha$ and $\beta$ in the given kinematic ranges when averaged over $\eta$ or $p_T$. The ratios involving $\Delta \gamma_{112}$ and $\Delta \gamma_{123}$ are again found to be similar differentially for all three variables in both pPb and PbPb collisions. This observation further supports a common origin of $\Delta \gamma_{112}$ and $\Delta \gamma_{123}$ from charge-dependent two-particle correlations coupled with the anisotropic flow.

### B. Event shape engineering

To explore directly the background scenario in Eq. (3) in terms of a linear dependence on $v_2$ for the $\gamma_{112}$ correlator, results based on the ESE analysis are presented in this section.

The SS and OS three-particle correlators $\gamma_{112}$ averaged over $|\Delta \eta| < 1.6$, are shown as a function of $v_2$ (evaluated as the average $v_2$ value for each corresponding $d_2$ event class in Fig. 11) for the multiplicity range $185 \leq N_{trk}^{\text{offline}} < 250$ in pPb collisions at $\sqrt{s_{NN}} = 8.16$ TeV (upper) and PbPb collisions at 5.02 TeV (lower). The pPb results are obtained with particle $c$ from the Pb- and $p$-going sides separately.

Both SS and OS $\gamma_{112}$ correlators in both pPb (both beam directions for particle $c$) and PbPb collisions show a dependence on $v_2$. A clear linear dependence on the $v_2$ value is not seen for any of the SS and OS correlators studied.

Similar to the analysis in Sec. VA, the difference between OS and SS correlators is taken in order to eliminate the charge-independent sources of the correlators. The results, averaged over $|\Delta \eta| < 1.6$, are shown in Fig. 12 (upper), as a function of $v_2$ evaluated in each $d_2$ class, for the multiplicity range $185 \leq N_{trk}^{\text{offline}} < 250$ in pPb collisions at $\sqrt{s_{NN}} = 8.16$ TeV and PbPb collisions at 5.02 TeV. The results obtained in each centrality class of PbPb collisions at 5.02 TeV are also presented in Fig. 12 (lower). The lines are linear fits to the
data,

$$\Delta \gamma_{112} = a v_2 + b,$$

(15)

where the first term corresponds to the $v_2$-dependent background contribution with the slope parameter $a$ equal to $\kappa_2 \Delta \delta$ [from Eq. (3)], which is assumed to be $v_2$ independent. The intercept parameter $b$ denotes the $v_2$-independent contribution (when linearly extrapolating to $v_2 = 0$) in the $\gamma_{112}$ correlator. In particular, the CME contribution to the $\Delta \gamma_{112}$ is expected to be largely $v_2$ independent within narrow multiplicity (centrality) ranges, the $b$ parameter may provide an indication to a possible observation of the CME, or set an upper limit on the CME contribution.

As shown in Fig. 12, for both $p$Pb and PbPb collisions in each multiplicity or centrality range, a clear linear dependence of the $\Delta \gamma_{112}$ correlator as a function of $v_2$ is observed. Fitted by a linear function, the intercept parameter $b$ can be extracted. A one standard deviation uncertainty band is also shown for the linear fit. Taking the statistical uncertainties into account, the values of $b$ are found to be nonzero for multiplicity range $185 \leq N_{\text{off}} < 250$ in $p$Pb and 60–70% centrality in PbPb collisions.

Observing a nonzero intercept $b$ from Fig. 12 may or may not lead to a conclusion of a finite CME signal, as an assumption is made for the background contribution term, namely that $\Delta \delta$ is independent of $v_2$. To check this assumption explicitly, the $\Delta \delta$ correlator is shown in Fig. 13 as a function of $v_2$ in different multiplicity and centrality ranges in $p$Pb (upper) and PbPb (lower) collisions. It is observed that the value of $\Delta \delta$ remains largely constant as a function of $v_2$ in low- or intermediate-$q_2$ classes, but starts rising as $v_2$ increases in high-$q_2$ classes. The multiplicity, within a centrality or multiplicity range, decreases slightly with increasing $q_2$, which qualitatively could contribute to the rising $\Delta \delta$ due to a multiplicity dilution effect. However, this is only found to be true for PbPb collisions, but not for $p$Pb collisions. The other reason may be related to larger jetlike correlations selected by requiring large $q_2$ values. Events with higher multiplicities show a weaker dependence on $v_2$ than those with lower multiplicities, which is consistent with

FIG. 11. The SS and OS three-particle correlators $\gamma_{112}$ averaged over $|\Delta \eta| < 1.6$ as a function of $v_2$ (evaluated as the average $v_2$ value for each corresponding $q_2$ event class) for the multiplicity range $185 \leq N_{\text{off}} < 250$ in $p$Pb collisions at $\sqrt{s_{NN}} = 8.16$ TeV (upper), and PbPb collisions at 5.02 TeV (lower). The $p$Pb results are obtained with particle $c$ from $p$- and $p$-going sides separately. Statistical and systematic uncertainties are indicated by the error bars and shaded regions, respectively.

FIG. 12. The difference of the OS and SS three-particle correlators $\gamma_{112}$ averaged over $|\Delta \eta| < 1.6$ as a function of $v_2$ evaluated in each $q_2$ class, for the multiplicity range $185 \leq N_{\text{off}} < 250$ in $p$Pb collisions at $\sqrt{s_{NN}} = 8.16$ TeV and PbPb collisions at 5.02 TeV (lower), and for different centrality classes in PbPb collisions at 5.02 TeV (lower). Statistical and systematic uncertainties are indicated by the error bars and shaded regions, respectively. A one standard deviation uncertainty from the fit is also shown.
the expectation that short-range jetlike correlations are stronger in peripheral events. Because of the possible bias towards larger jetlike correlations at higher \( q_T \) from the ESE technique, the \( v_2 \) dependence of \( \Delta \delta \) is hard to completely eliminate. This presents a challenge to the interpretation of the intercept values from the linear fits in Fig. 12.

In order to avoid the issue of \( \Delta \delta \) being dependent on \( v_2 \), the ratio \( \Delta \gamma_{12}/\Delta \delta \) as a function of \( v_2 \) is shown in Fig. 14 for different multiplicity ranges in \( pPb \) collisions at \( \sqrt{s_{NN}} = 8.16 \text{ TeV} \) (upper) and for different centrality classes in \( PbPb \) collisions at 5.02 TeV (lower). Particularly in the scenario of a pure \( v_2 \)-dependent background, the ratio \( \Delta \gamma_{12}/\Delta \delta \) is expected to be proportional to \( v_2 \). A linear function is fitted again using

\[
\frac{\Delta \gamma_{12}}{\Delta \delta} = a_{\text{norm}} v_2 + b_{\text{norm}}. \tag{16}
\]

Here, comparing to the intercept parameter \( b \) in Eq. (15), the \( b_{\text{norm}} \) parameter is equivalent to \( b \) scaled by the \( \Delta \delta \) factor. The fitted linear slope and intercept parameters, \( a_{\text{norm}} \) and \( b_{\text{norm}} \), are summarized in Tables II and III in \( N_{\text{offline}} \) and centrality classes for \( pPb \) and \( PbPb \) collisions, respectively.

The values of the intercept parameter \( b_{\text{norm}} \) are shown as a function of event multiplicity in Fig. 15 (upper), for both \( pPb \) and \( PbPb \) collisions. The \( \pm 1\sigma \) and \( \pm 2\sigma \) systematic uncertainties are indicated by the error bars and shaded regions, respectively.
TABLE II. The summary of slope and intercept parameter $a_{norm}$ and $b_{norm}$ for different $N_{trk}^{\text{offline}}$ classes in $p$Pb collisions, and the goodness of fit $\chi^2$ per degree of freedom (ndf). The statistical and systematic uncertainties are shown after the central values, respectively.

<table>
<thead>
<tr>
<th>$N_{trk}^{\text{offline}}$</th>
<th>$a_{norm}$</th>
<th>$b_{norm}$</th>
<th>$\chi^2$/ndf</th>
</tr>
</thead>
<tbody>
<tr>
<td>120–150</td>
<td>$1.13 \pm 0.24 \pm 0.14$</td>
<td>$0.048 \pm 0.019 \pm 0.012$</td>
<td>16.3/8</td>
</tr>
<tr>
<td>150–185</td>
<td>$1.13 \pm 0.19 \pm 0.04$</td>
<td>$0.047 \pm 0.016 \pm 0.008$</td>
<td>4.9/8</td>
</tr>
<tr>
<td>185–250</td>
<td>$1.69 \pm 0.06 \pm 0.01$</td>
<td>$-0.0009 \pm 0.0050 \pm 0.0078$</td>
<td>4.5/8</td>
</tr>
<tr>
<td>250–300</td>
<td>$1.83 \pm 0.13 \pm 0.15$</td>
<td>$-0.015 \pm 0.011 \pm 0.016$</td>
<td>8.1/8</td>
</tr>
</tbody>
</table>

The observed CME signal may be reduced as $v_2$ decreases for small $v_2$ values (e.g., $<6\%$), due to a weaker correlation between magnetic field and event-plane orientations as a result of initial-state fluctuations. Depending on specific models of initial-state fluctuations, the upper limits obtained in this paper may increase relatively by about 20%, although still well within a few % level. On the other hand, covering a wide range of $v_2$ values in this analysis (6–15%), the $v_2$ dependence of the observed CME signal is minimized to the largest extent, especially for more central events. The data also rule out any significant nonlinear $v_2$ dependence of the observed CME signal, as suggested by Ref. [46]. Therefore, the high-precision data presented in this paper indicate that the charge-dependent three-particle azimuthal correlations in $p$Pb and PbPb collisions are consistent with a $v_2$-dependent background-only scenario, posing a significant challenge to the search for the CME in heavy ion collisions using three-particle azimuthal correlations.

VI. SUMMARY

Charge-dependent azimuthal correlations of same- and opposite-sign (SS and OS) pairs with respect to the second- and third-order event planes have been studied in $p$Pb collisions at $\sqrt{s_{NN}} = 8.16$ TeV and PbPb collisions at 5.02 TeV by the CMS experiment at the LHC. The correlations are extracted via three-particle correlators as functions of pseudorapidity difference, transverse momentum difference, and $p_T$ average of SS and OS particle pairs, in various multiplicity or centrality ranges of the collisions. The differences in correlations between OS and SS particles with respect to both second- and third-order event planes as functions of $\Delta \eta$ and multiplicity are found to agree for $p$Pb and PbPb collisions, indicating a common underlying mechanism for the two systems. Dividing the OS and SS difference of the three-particle correlator by the product of the $v_n$ harmonic of the corresponding order

TABLE III. The summary of slope and intercept parameter $a_{norm}$ and $b_{norm}$ for different centrality classes in PbPb collisions, and the goodness of fit $\chi^2$ per degree of freedom (ndf). The statistical and systematic uncertainties are shown after the central values, respectively.

<table>
<thead>
<tr>
<th>Centrality</th>
<th>$a_{norm}$</th>
<th>$b_{norm}$</th>
<th>$\chi^2$/ndf</th>
</tr>
</thead>
<tbody>
<tr>
<td>60–70%</td>
<td>$1.85 \pm 0.17 \pm 0.21$</td>
<td>$0.003 \pm 0.017 \pm 0.023$</td>
<td>12.3/9</td>
</tr>
<tr>
<td>50–60%</td>
<td>$1.75 \pm 0.04 \pm 0.01$</td>
<td>$0.002 \pm 0.004 \pm 0.010$</td>
<td>11.8/9</td>
</tr>
<tr>
<td>45–50%</td>
<td>$1.74 \pm 0.04 \pm 0.03$</td>
<td>$0.000 \pm 0.005 \pm 0.011$</td>
<td>8.4/9</td>
</tr>
<tr>
<td>40–45%</td>
<td>$1.59 \pm 0.03 \pm 0.01$</td>
<td>$0.012 \pm 0.003 \pm 0.011$</td>
<td>9.1/9</td>
</tr>
<tr>
<td>35–40%</td>
<td>$1.68 \pm 0.03 \pm 0.01$</td>
<td>$-0.001 \pm 0.003 \pm 0.010$</td>
<td>15.1/9</td>
</tr>
<tr>
<td>30–35%</td>
<td>$1.67 \pm 0.04 \pm 0.01$</td>
<td>$-0.0026 \pm 0.0036 \pm 0.0095$</td>
<td>6.9/9</td>
</tr>
</tbody>
</table>
and the difference of the two-particle correlator, the ratios are found to be similar for the second- and third-order event planes, and show a weak dependence on event multiplicity. These observations support a scenario in which the charge-dependent three-particle correlator is predominantly a consequence of charge-dependent two-particle correlations coupled to an anisotropic flow signal.

To establish the relation between the three-particle correlator and anisotropic flow harmonic in detail, an event shape engineering technique is applied. A linear relation for the ratio of three- to two-particle correlator difference as a function of $v_2$ is observed, which extrapolates to an intercept that is consistent with zero within uncertainties for most of multiplicities. An upper limit on the $v_2$-independent fraction of the three-particle correlator, or the possible CME signal contribution (assumed independent of $v_2$ within the same narrow multiplicity or centrality range), is estimated to be 13% for $p\bar{p}$ data and 7% for PbPb data at a 95% confidence level. The data presented in this paper provide new stringent constraints on the nature of the background contribution to the charge-dependent azimuthal correlations, and establish a new baseline for the search for the chiral magnetic effect in heavy ion collisions.

ACKNOWLEDGMENTS

We congratulate our colleagues in the CERN accelerator departments for the excellent performance of the LHC and thank the technical and administrative staffs at CERN and at other CMS institutes for their contributions to the success of the CMS effort. In addition, we gratefully acknowledge the computing centers and personnel of the Worldwide LHC Computing Grid for delivering so effectively the computing infrastructure essential to our analyses. Finally, we acknowledge the enduring support for the construction and operation of the LHC and the CMS detector provided by the following funding agencies: BMWF and FWO (Austria); FNRS and FWO (Belgium); CNPq, CAPES, FAPERJ, and FAPESP (Brazil); MES (Bulgaria); CERN; CAS, MoST, and NSFC (China); COLCIENCIAS (Colombia); MSES and CSF (Croatia); RPF (Cyprus); SENESCYT (Ecuador); MoER, ERC IUT, and ERDF (Estonia); Academy of Finland, MEC, and HIP (Finland); CEA and CNRS/IN2P3 (France); BMBF, DFG, and HGF (Germany); GSRT (Greece); OTKA and NIH (Hungary); DAE and DST (India); IPM (Iran); SFI (Ireland); INFN (Italy); MSIP and NRF (Republic of Korea); LAS (Lithuania); MOE and UM (Malaysia); BUAP, CINVESTAV, CONACYT, LNS, SEP, and UASLP-FAI (Mexico); MBIE (New Zealand); PAEC (Pakistan); MSHE and NSC (Poland); FCT (Portugal); JINR (Dubna); MON, RosAtom, RAS, RFBR and RAEP (Russia); MESTD (Serbia); SEIDI, CPAN, PCTI and FEDER (Spain);
FIG. 17. The $v_{2,c}$ using particle $c$ from HF+ and HF− data are shown as a function of $v_2$ in the tracker region ($|\eta| < 2.4$) in PbPb collisions at $\sqrt{s_{NN}} = 5.02$ TeV.

Swiss Funding Agencies (Switzerland); MST (Taipei); ThEP-Center, IPST, STAR, and NSTDA (Thailand); TUBITAK and TAEK (Turkey); NASU and SFFR (Ukraine); STFC (United Kingdom); DOE and NSF (USA). Individuals have received support from the Marie-Curie program and the European Research Council and a Horizon 2020 grant, Contract No. 675440 (European Union); the Leventis Foundation; the A. P.

FIG. 18. The $v_{2,c}$ using particle $c$ from the Pb-going side of the HF (4.4 < $\eta$ < 5.0) data are shown as a function of $v_2$ in the tracker region ($|\eta| < 2.4$) in pPb collisions at $\sqrt{s_{NN}} = 8.16$ TeV.

Sloan Foundation; the Alexander von Humboldt Foundation; the Belgian Federal Science Policy Office; the Fonds pour la Formation à la Recherche dans l’Industrie et dans l’Agriculture (FRIA-Belgium); the Agentschap voor Innovatie door Wetenschap en Technologie (IWT-Belgium); the Ministry of Education, Youth and Sports (MEYS) of the Czech Republic; the Council of Science and Industrial Research, India; the HOMING PLUS program of the Foundation for Polish Science, cofinanced from European Union, Regional Development Fund, the Mobility Plus program of the Ministry of Science and Higher Education, the National Science Center (Poland), Harmonia Contract No. 2014/14/M/ST2/00428, Opus Contracts No. 2014/13/B/ST2/02543, No. 2014/15/B/ST2/03998, and No. 2015/19/B/ST2/02861, Sonata-bis Contract No. 2012/07/E/ST2/01406; the National Priorities Research Program by Qatar National Research Fund; the Programa de Fortalecimiento Institucional del Consejo Nacional de Ciencia y Tecnología (CONICYT) de Chile; the Fondation pour la Formation à la Recherche dans l’Industrie et dans l’Agriculture (FRIA-Belgium); the Agentschap voor Innovatie door Wetenschap en Technologie (IWT-Belgium); the Ministry of Education, Youth and Sports (MEYS) of the Czech Republic; the Council of Science and Industrial Research, India; the HOMING PLUS program of the Foundation for Polish Science, cofinanced from European Union, Regional Development Fund, the Mobility Plus program of the Ministry of Science and Higher Education, the National Science Center (Poland), Harmonia Contract No. 2014/14/M/ST2/00428, Opus Contracts No. 2014/13/B/ST2/02543, No. 2014/15/B/ST2/03998, and No. 2015/19/B/ST2/02861, Sonata-bis Contract No. 2012/07/E/ST2/01406; the National Priorities Research Program by Qatar National Research Fund; the Programa Clarín-COOFUND del Principado de Asturias; the Thalis and Aristeia programs cofinanced by EU-ESF and the Greek NSRF; the Rachadapisek Sompot Fund for a postdoctoral fellowship, Chulalongkorn University and the Chulalongkorn Academic into its 2nd Century Project Advancement Project (Thailand); and the Welch Foundation, Contract No. C-1845.
APPENDIX A: GENERAL RELATION OF $v_n$ HARMONICS AND TWO- AND THREE-PARTICLE AZIMUTHAL CORRELATIONS

In Sec. I, Eq. (5) can be derived in a way similar to Eq. (3), with details which can be found in Ref. [24]. Here, a general derivation of Eq. (5) for all higher-order-harmonic correlators is given.

Similar to Eq. (40) in Ref. [24], the general relation between the $n$th order anisotropy harmonic $v_n$ and the three-particle correlator with respect to the $n$th order event plane can be
FIG. 21. The SS and OS three-particle correlators $\gamma_{112}$ (upper) and $\gamma_{123}$ (middle) and two-particle correlator $\delta$ (lower) as a function of $|\Delta p_T|$ for four multiplicity ranges in $p$Pb collisions at $\sqrt{s_{NN}} = 8.16$ TeV (left) and PbPb collisions at 5.02 TeV (right) collisions. The $p$Pb results obtained with particle $c$ in Pb-going (solid markers) and $p$-going (open markers) sides are shown separately. The SS and OS two-particle correlators are denoted by different markers for both $p$Pb and PbPb collisions. Statistical and systematic uncertainties are indicated by the error bars and shaded regions, respectively.

derived starting from

$$\gamma_{1,n-1,n} = \langle \cos(\phi_a + (n-1)\phi_b - n\Psi_n) \rangle$$

$$= \frac{\int \rho_2 \cos[\phi_a + (n-1)\phi_b - n\Psi_n] d\phi_a d\phi_b dx_\alpha dx_\beta}{\int \rho_2 d\phi_a d\phi_b dx_\alpha dx_\beta},$$

(A1)

where $x$ denotes ($p_T, \eta$) and $dx = p_T dp_T d\eta$. $\rho_2$ is the two-particle pair density distribution, which can be expressed...
in terms of the single-particle density distribution and its underlying two-particle correlation function (see Sec. 2 in Ref. [24]),

$$\rho_2 = \rho(\phi_a, x_a)\rho(\phi_b, x_b)[1 + C(\phi_a, \phi_b, x_a, x_b)]. \quad (A2)$$

In the presence of collective anisotropy flow, the single-particle azimuthal distribution can be expressed in terms of a Fourier series with respect to the event plane of the corresponding order,

$$\rho(\phi, x) = \frac{\rho_0(x)}{2\pi} \left[ 1 + \sum_{n=1}^{\infty} n v_n(x) \cos n(\phi - \Psi_n) \right], \quad (A3)$$

where $\rho_0(x)$ depends on $p_T$ and $\eta$ only.
FIG. 23. The SS and OS three-particle correlators $\gamma_{123}$ (upper) and $\gamma_{123}$ (middle) and two-particle correlator $\delta$ (lower), as a function of $|\Delta\eta|$ for five centrality classes in PbPb collisions at 5.02 TeV. The SS and OS two-particle correlators are denoted by different markers. Statistical and systematic uncertainties are indicated by the error bars and shaded regions, respectively.

The two-particle correlation function $C$ describes intrinsic correlations that are insensitive to the event plane $\Psi_\alpha$, but only involve azimuthal angle difference $\Delta \phi = \phi_\alpha - \phi_\beta$. It can be also expanded in Fourier series [24],

$$C(\Delta \phi, x_\alpha, x_\beta) = \sum_{n=1}^{\infty} a_n(x_\alpha, x_\beta) \cos(n \Delta \phi), \quad (A4)$$

where $a_n(x_\alpha, x_\beta)$ is the two-particle Fourier coefficient. By definition, $a_1(x_\alpha, x_\beta)$ is equal to the two-particle correlator $\delta(x_\alpha, x_\beta)$, introduced in Sec. I, as a function of $x_\alpha$ and $x_\beta$ (i.e., $p_T$ and $\eta$ of both particles).

Therefore, we substitute Eqs. (A4) and (A2) into (A1) and obtain

$$\gamma_{1,n-1,n} = \frac{1}{2N^2} \int \rho_0(x_\alpha)\rho_0(x_\beta)a_1(x_\alpha, x_\beta) \times [\nu_n(x_\alpha) + \nu_n(x_\beta)] dx_\alpha dx_\beta$$
FIG. 24. The SS and OS three-particle correlators $\gamma_{112}$ (upper) and $\gamma_{123}$ (middle) and two-particle correlator $\delta$ (lower) as a function of $|\Delta p_T|$ for five centrality classes in PbPb collisions at $\sqrt{s_{NN}} = 5.02$ TeV. The SS and OS two-particle correlators are denoted by different markers. Statistical and systematic uncertainties are indicated by the error bars and shaded regions, respectively.

\[
\gamma_{112,n-1,n} = \frac{1}{2N^2} \int \rho_0(x_{\alpha})\rho_0(x_{\beta})\delta(\mathbf{x}_{\alpha},\mathbf{x}_{\beta})
\times [v_n(x_{\alpha}) + v_n(x_{\beta})]dx_{\alpha} \, dx_{\beta}, \tag{A5}
\]

where $N = \int \rho_0(x)dx$. This is the general equation explaining why a nonzero two-particle correlation $\delta(\mathbf{x}_{\alpha},\mathbf{x}_{\beta})$ plus an anisotropy flow of $n$th order $v_n(x)$ contribute to the three-particle correlator, $\gamma_{112,n-1,n}$.

Therefore, this general form of $\gamma_{112,n-1,n}$ can be applied to any order $n$ and decomposed into the two-particle correlator $\delta$ and the $n$th order harmonic $v_n$, where $n = 2$ and 3 are studied in detail in Sec. V A.
FIG. 25. The SS and OS three-particle correlators $\gamma_{112}$ (upper) and $\gamma_{123}$ (middle) and two-particle correlator $\delta$ (lower) as a function of $p_T$ for five centrality classes in PbPb collisions at $\sqrt{s_{NN}} = 5.02$ TeV. The SS and OS two-particle correlators are denoted by different markers. Statistical and systematic uncertainties are indicated by the error bars and shaded regions, respectively.

APPENDIX B: SUPPORTING RESULTS OF THE EVENT SHAPE ENGINEERING METHOD

As stated in Sec. IV B, the $Q_2$ vector is calculated using one side of the HF detector within the $\eta$ range of 3–5 units. The default result in Sec. V B presents the $\Delta \gamma_{112}$ as a function of $v_2$, where the particle $c$ in the $\gamma_{112}$ correlator corresponds to the $\eta$ range $-5.0$ to $-4.4$. However, the results are found to be independent of where the particle $c$ is reconstructed, as it is shown in Fig. 16.

In Figs. 17 and 18, the denominators of Eq. (7), $v_{2,c}$, for different $Q_2$ classes with respect to HF+ and HF− in PbPb collisions at $\sqrt{s_{NN}} = 5.02$ TeV, and the Pb-going side of the HF in $p$Pb collisions at 8.16 TeV, are shown as a function of $v_2$ in the tracker region. Here $v_{2,c}$ is a measure of elliptic anisotropy of the transverse energy registered in the HF detectors without
being corrected to the particle-level elliptic flow. It serves as the resolution correction factor when deriving the three-particle correlators or the $v_2$ values in the tracker region using the scalar-product method.

In Fig. 19, the average $N_{\text{offline}}$ is shown as a function of $v_2$ in different multiplicity and centrality ranges in pPb (upper) and PbPb collisions (lower), respectively. The average $N_{\text{offline}}$ is found to be weakly dependent on $v_2$, but with a slight decreasing trend as $v_2$ increases. Similar to Fig. 13, the effect at low multiplicities is stronger than that at high multiplicities. Overall, this effect is negligible for the results shown in Sec. VB.

APPENDIX C: THREE- AND TWO-PARTICLE CORRELATOR AS FUNCTIONS OF DIFFERENTIAL VARIABLES IN DIFFERENT MULTIPLICITY AND CENTRALITY CLASSES

The figures in Appendix C show the $\gamma_{12}, \gamma_{13},$ and $\delta$ correlators as a function of $|\Delta p_T|$, $|\Delta \eta|$, and $p_T$ in pPb collisions at $\sqrt{s_{NN}} = 8.16$ TeV and PbPb collisions at 5.02 TeV. In pPb and PbPb collisions, the results are shown for multiplicity ranges $N_{\text{offline}} = [120, 150], [150, 185], [185, 250], \text{ and } [250, 300]$ in Figs. 20–22. In PbPb collisions, the results are also shown for five centrality classes from 30–80% in Figs. 23–25.


[38] ALICE Collaboration, Event shape engineering for inclusive spectra and elliptic flow in Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV, Phys. Rev. C 93, 034916 (2016).


[46] ALICE Collaboration, Constraining the magnitude of the Chiral Magnetic Effect with Event Shape Engineering in Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV, arXiv:1709.04723.


A. M. SIRUNYAN AN


[38] ALICE Collaboration, Event shape engineering for inclusive spectra and elliptic flow in Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV, Phys. Rev. C 93, 034916 (2016).


[46] ALICE Collaboration, Constraining the magnitude of the Chiral Magnetic Effect with Event Shape Engineering in Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV, arXiv:1709.04723.
University of Cyprus, Nicosia, Cyprus
Charles University, Prague, Czech Republic
Universidad San Francisco de Quito, Quito, Ecuador
Academy of Scientific Research and Technology of the Arab Republic of Egypt, Egyptian Network of High Energy Physics, Cairo, Egypt
National Institute of Chemical Physics and Biophysics, Tallinn, Estonia
Department of Physics, University of Helsinki, Helsinki, Finland
Helsinki Institute of Physics, Helsinki, Finland
Lappeenranta University of Technology, Lappeenranta, Finland
IRFU, CEA, Université Paris-Saclay, Gif-sur-Yvette, France
Laboratoire Leprince-Ringuet, Ecole polytechnique, CNRS/IN2P3, Université Paris-Saclay, Palaiseau, France
Université de Strasbourg, CNRS, IPHC UMR 7178, F-67000 Strasbourg, France
Centre de Calcul de l’Institut National de Physique Nucleaire et de Physique des Particules, CNRS/IN2P3, Villeurbanne, France
Université de Lyon, Université Claude Bernard Lyon 1, CNRS-IN2P3, Institut de Physique Nucléaire de Lyon, Villeurbanne, France
Georgian Technical University, Tbilisi, Georgia
Tbilisi State University, Tbilisi, Georgia
RWTH Aachen University, I. Physikalisches Institut, Aachen, Germany
RWTH Aachen University, III. Physikalisches Institut A, Aachen, Germany
RWTH Aachen University, III. Physikalisches Institut B, Aachen, Germany
Deutsches Elektronen-Synchrotron, Hamburg, Germany
University of Hamburg, Hamburg, Germany
Institut für Experimentelle Kernphysik, Karlsruhe, Germany
Institute of Nuclear and Particle Physics (INPP), NCSR Demokritos, Aghia Paraskevi, Greece
National and Kapodistrian University of Athens, Athens, Greece
National Technical University of Athens, Athens, Greece
University of Ioannina, Ioannina, Greece
MTA-ELTE Lendület CMS Particle and Nuclear Physics Group, Eötvös Loránd University, Budapest, Hungary
Wigner Research Centre for Physics, Budapest, Hungary
Institute of Nuclear Research ATOMKI, Debrecen, Hungary
Institute of Physics, University of Debrecen, Debrecen, Hungary
Indian Institute of Science (IISc), Bangalore, India
National Institute of Science Education and Research, Bhubaneswar, India
Panjab University, Chandigarh, India
University of Delhi, Delhi, India
Saha Institute of Nuclear Physics, HBNI, Kolkata, India
Indian Institute of Technology Madras, Madras, India
Bhabha Atomic Research Centre, Mumbai, India
Tata Institute of Fundamental Research, Mumbai, India
Indian Institute of Science Education and Research (IISER), Pune, India
Institute for Research in Fundamental Sciences (IPM), Tehran, Iran
University College Dublin, Dublin, Ireland
INFN Sezione di Bari, Bari, Italy
Université di Bari, Bari, Italy
Politecnico di Bari, Bari, Italy
INFN Sezione di Bologna, Bologna, Italy
Università di Bologna, Bologna, Italy
INFN Sezione di Catania, Catania, Italy
Università di Catania, Catania, Italy
INFN Sezione di Firenze, Firenze, Italy
Università di Firenze, Firenze, Italy
INFN Laboratori Nazionali di Frascati, Frascati, Italy
INFN Sezione di Genova, Genova, Italy
Università di Genova, Genova, Italy
INFN Sezione di Milano-Bicocca, Milano, Italy
Università di Milano-Bicocca, Milano, Italy
INFN Sezione di Napoli, Roma, Italy
Università di Napoli ‘Federico II’, Roma, Italy
70c Napoli, Italy, Università della Basilicata, Italy, Roma, Italy
70d Potenza Università G. Marconi, Roma, Italy
71c INFN Sezione di Padova, Trento, Italy
71b Università di Padova, Trento, Italy
71c Padova, Italy, Università di Trento, Trento, Italy
72a INFN Sezione di Pavia, Pavia, Italy
72b Università di Pavia, Pavia, Italy
73a INFN Sezione di Perugia, Perugia, Italy
73b Università di Perugia, Perugia, Italy
74a INFN Sezione di Pisa, Pisa, Italy
74b Università di Pisa, Pisa, Italy
74c Scuola Normale Superiore di Pisa, Pisa, Italy
75a INFN Sezione di Roma, Rome, Italy
75b Sapienza Università di Roma, Rome, Italy
76a INFN Sezione di Torino, Novara, Italy
76b Università di Torino, Novara, Italy
76c Torino, Italy, Università del Piemonte Orientale, Novara, Italy
77a INFN Sezione di Trieste, Trieste, Italy
77b Università di Trieste, Trieste, Italy
78 Kyungpook National University, Daegu, Korea
79 Chonbuk National University, Jeonju, Korea
80 Chonnam National University, Institute for Universe and Elementary Particles, Kwangju, Korea
81 Hanyang University, Seoul, Korea
82 Korea University, Seoul, Korea
83 Seoul National University, Seoul, Korea
84 Seoul University, Seoul, Korea
85 Sungkyunkwan University, Suwon, Korea
86 Vilnius University, Vilnius, Lithuania
87 National Centre for Particle Physics, Universiti Malaya, Kuala Lumpur, Malaysia
88 Centro de Investigacion y de Estudios Avanzados del IPN, Mexico City, Mexico
89 Universidad Iberoamericana, Mexico City, Mexico
90 Benemerita Universidad Autonoma de Puebla, Puebla, Mexico
91 Universidad Autónoma de San Luis Potosí, San Luis Potosí, Mexico
92 University of Auckland, Auckland, New Zealand
93 University of Canterbury, Christchurch, New Zealand
94 National Centre for Physics, Quaid-I-Azam University, Islamabad, Pakistan
95 National Centre for Nuclear Research, Swierk, Poland
96 Institute of Experimental Physics, Faculty of Physics, University of Warsaw, Warsaw, Poland
97 Laboratório de Instrumentação e Física Experimental de Partículas, Lisboa, Portugal
98 Joint Institute for Nuclear Research, Dubna, Russia
99 Petersburg Nuclear Physics Institute, Gatchina (St. Petersburg), Russia
100 Institute for Nuclear Research, Moscow, Russia
101 Institute for Theoretical and Experimental Physics, Moscow, Russia
102 Moscow Institute of Physics and Technology, Moscow, Russia
103 National Research Nuclear University ‘Moscow Engineering Physics Institute’ (MEPhI), Moscow, Russia
104 P.N. Lebedev Physical Institute, Moscow, Russia
105 Skobeltsyn Institute of Nuclear Physics, Lomonosov Moscow State University, Moscow, Russia
106 Novosibirsk State University (NSU), Novosibirsk, Russia
107 State Research Center of Russian Federation, Institute for High Energy Physics, Protvino, Russia
108 University of Belgrade, Faculty of Physics and Vinca Institute of Nuclear Sciences, Belgrade, Serbia
109 Centro de Investigaciones Energéticas, Medioambientales y Tecnológicas (CIEMAT), Madrid, Spain
110 Universidad Autónoma de Madrid, Madrid, Spain
111 Universidad de Oviedo, Oviedo, Spain
112 Instituto de Física de Cantabria (IFCA), CSIC-Universidad de Cantabria, Santander, Spain
113 CERN, European Organization for Nuclear Research, Geneva, Switzerland
114 Paul Scherrer Institut, Villigen, Switzerland
115 Institute for Particle Physics, ETH Zurich, Zurich, Switzerland
116 Universität Zürich, Zurich, Switzerland
117 National Central University, Chung-Li, Taiwan
Necmettin Erbakan University, Konya, Turkey.
Marmara University, Istanbul, Turkey.
Kafkas University, Kars, Turkey.
Istanbul Bilgi University, Istanbul, Turkey.
Rutherford Appleton Laboratory, Didcot, United Kingdom.
School of Physics and Astronomy, University of Southampton, Southampton, United Kingdom.
Instituto de Astrofísica de Canarias, La Laguna, Spain.
Utah Valley University, Orem, USA.
Beykent University, Istanbul, Turkey.
Bingol University, Bingol, Turkey.
Erzincan University, Erzincan, Turkey.
Sinop University, Sinop, Turkey.
Mimar Sinan University, Istanbul, Istanbul, Turkey.
Institute for Nuclear Research, Moscow, Russia.
Texas A&M University at Qatar, Doha, Qatar.
Kyungpook National University, Daegu, Korea.