Multi-fidelity Gaussian Process Regression for Propeller Optimisation Under Uncertainty

Péter Zénó Korondi, Mariapia Marchi, Lucia Parussini, Carlo Poloni
peterzeno.korondi@phd.units.it

Introduction
The motivation of this research is to optimise the propeller of a small-scale electrical aircraft under uncertainty. The expensive performance evaluation prohibits the application of standard optimisation techniques and the direct calculation of statistical measures. This motivates the use of cheap low-fidelity simulations to obtain more information about the unexplored locations of the input space. The information stemming from the low- and high-fidelity simulations are fused together with multi-fidelity Gaussian Process Regression to build an accurate surrogate model despite the low number of high-fidelity simulations. The proposed surrogate-based optimisation workflow allows us to efficiently carry out an optimisation problem which otherwise would be impracticable.

Optimisation Workflow
The proposed surrogate-based optimisation workflow allows us to efficiently carry out an optimisation problem which otherwise would be impracticable. Multi-fidelity surrogate techniques require high correlation between the fidelities. Fortunately, in aerospace engineering even low-fidelity models (Lifting-line Theory, Blade Element Momentum Theory) are well calibrated and the correlation with high-fidelity Navier-Stokes Solvers is good. This allows us to obtain valuable information on the performance of a design at a cheaper cost.

Conclusion
Multi-fidelity optimisation techniques require good cross-correlation between fidelity levels. Multi-fidelity surrogate techniques can help construct accurate surrogates when only sparse high-fidelity samples are available. An acquisition function calculating the expected variance reduction can efficiently choose where to sample next. The separate design and probability space modelling approach facilitates the use of appropriate surrogate techniques in each space. Multi-fidelity techniques in aerospace applications can increase efficiency as well-calibrated low-fidelity formulae are available.

Propeller Performance Solvers

<table>
<thead>
<tr>
<th>Solver</th>
<th>Speed</th>
<th>Samples</th>
<th>Accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td>CFD</td>
<td>~3.5 h</td>
<td>few</td>
<td>highest</td>
</tr>
<tr>
<td>XROTOR2</td>
<td>~30 s</td>
<td>some</td>
<td>intermediate</td>
</tr>
<tr>
<td>BEMT3</td>
<td>~0.1 s</td>
<td>many</td>
<td>lowest</td>
</tr>
</tbody>
</table>

1 Navier-Stokes Solver (SU2)
2 Lifting-line Theory
3 Blade Element Momentum Theory

Uncertainty Quantification
The probability space is modelled by Polynomial Chaos Expansion (PCE). The PCE is a very efficient technique and depending on its polynomial family it can model the probability space of various probability distributions.

\[ f(x) \approx \sum_{i=1}^{k} \alpha_i p_i(x), \]

Multi-fidelity Gaussian Process Regression
A complex process can be approximated by the sum of two processes. The low-fidelity process can be modelled accurately as many data can be generated. The difference term is typically a less complex process.

\[ Z_i(x) = \rho_{i-1}(x)Z_{i-1}(x) + Z_k(x) \]
\[ \tilde{m}_i(x) = \rho_{i-1}\tilde{m}_{i-1}(x) + \tilde{m}_k(x) \]
\[ \tilde{s}_i^2(x) = \rho_{i-1}\tilde{s}_{i-1}^2(x) + \tilde{s}_k^2(x) \]

Acquisition Function
The fidelity level (l) is chosen which provides the highest scaled variance reduction:

\[ l = \arg \max_{\tilde{\delta}_l, LF,HF} \tilde{\delta}_l \]

where the \( \tilde{\delta}_l \) is defined as:

\[ \tilde{\delta}_{HF} = \tilde{s}_{HF}^2(x_{new})/\text{cost}_{HF} \]
\[ \tilde{\delta}_{LF} = \tilde{m}_{LF} - \tilde{s}_{LF}^2(x_{new})/\text{cost}_{LF} \]

Multi-fidelity Gaussian Process Regression

<table>
<thead>
<tr>
<th>Surrogate</th>
<th>Samples</th>
<th>Erroravg1</th>
<th>Erroropt2</th>
</tr>
</thead>
<tbody>
<tr>
<td>GPR</td>
<td>20</td>
<td>21.628</td>
<td>0.3949</td>
</tr>
<tr>
<td>MF-GPR</td>
<td>20(HF)</td>
<td>0.881</td>
<td>0.0002</td>
</tr>
</tbody>
</table>

1 Mean squared error
2 Absolute error of the location of the optimum

Publications

Results
The proposed surrogate-based optimisation algorithm achieved 6% improvement of the objective. The uncertain nature of the problem was captured with a nested probability modelling.

<table>
<thead>
<tr>
<th>Design</th>
<th>C_T</th>
<th>C_p</th>
<th>Objective</th>
</tr>
</thead>
<tbody>
<tr>
<td>baseline</td>
<td>2.281</td>
<td>3.367</td>
<td>1.476</td>
</tr>
<tr>
<td>optimum</td>
<td>1.173</td>
<td>1.624</td>
<td>1.384</td>
</tr>
</tbody>
</table>

1 Mean of the thrust coefficient
2 Mean of the power coefficient
3 Inverse of mean efficiency

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